Section 2-2 Describing Sets

A <u>set</u> is an undefined term. It is best defined as a collection of objects. The objects in a set are called <u>elements</u>. We typically use braces to denote a set and the symbol $\in$ to denote "is an element of."

For ex: $A = \{1, 2, 3, 4\}$, $4 \in A$

A is the set of #s 1, 2, 3, 4 — 4 is an element of set A

In order for a set to be useful, it must be <u>well-defined</u>.

$A = \{\text{whole \#s}\}$ well-defined
$B = \{\text{tall people}\}$ not well-defined

Roster notation $\{0, 1, 2, 3, 4, \ldots\}$

Set builder notation $\{x \mid x \geq 0, x \in W\}$

such that — x gr than or equal to 0 — x is a whole #

Two sets are equal (=) if and only if they contain exactly the same elements.

Two sets are equivalent (~) if and only if there exists a 1-1 correspondence between the 2 sets. (i.e., the 2 sets have the same # of elements.)

The number of elements in a set is called the cardinal number or cardinality. We denote the cardinality of a set with the notation $n(A)$.

True or False

1. All equal sets are equivalent. True

2. All equivalent sets are equal. False

A set with no elements, is said to be the empty set or null set.
We denote the empty set with $\{\}$ or $\emptyset$.

$\{\emptyset\}$ is not the empty set

ex 1 Given:

$A = \{1, 3, 5, 7\}$ $\quad$ $C = \{5, 1, 3, 7\}$

$B = \{2, 4, 6, 8\}$ $\quad$ $D = \{2, 4, 6\}$

True or False

a) $A = B$ False
b) $A \sim B$ True $n(A) = n(B)$
c) $A = C$ True
d) $A \sim C$ True
e) $B \sim C$ True $n(B) = n(C)$
f) $B = C$ False

ex 2 Give a real-world example of a set that is empty.

$D = \{$female presidents of the US$\}$

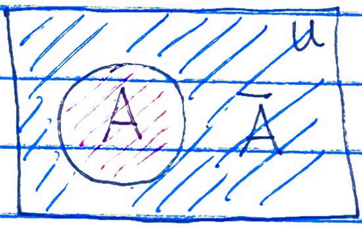
$E = \{$numbers that are both bigger than 10 and smaller than 2$\}$

$\emptyset = \{\ \}$

Be careful: $\{\emptyset\}$ is not empty neither is $\{0\}$.

The universe (or universal set), denoted $\mathcal{U}$, is the set of all elements being considered in a given problem.

The set of elements that are in the universe but not in set A is said to be the complement of set A, denoted $\overline{A}$.



For ex: $\mathcal{U} = \{\text{states in the US}\}$

$A = \{\text{states that start with A}\}$

then $\overline{A} = \{\text{states that do not start with A}\}$

Note: $\overline{\mathcal{U}} = \emptyset$ and $\overline{\emptyset} = \mathcal{U}$.

B is a subset of A, denoted $B \subseteq A$ if every element in B is also in A.

Note: If $A = B$ then $A \subseteq B$ and $B \subseteq A$

If $B \subseteq A$ and $B \neq A$, then B is a proper subset of A, denoted $B \subset A$.

ex 3 True or False

a) If $A \subseteq B$ then $A \subset B$. False

b) If $A \subset B$ then $A \subseteq B$. True

ex 4 Given: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

$A = \{2, 4, 6, 8, 10\}$ $B = \{2, 3, 5, 7\}$

a) What is $\bar{A}$? $\{1, 3, 5, 7, 9\}$

b) Is $B \subseteq A$? No $3, 5, 7 \notin A$

c) Is $A \sim B$? No $n(A) \neq n(B)$

d) What is $n(A)$? 5

e) Is $6 \in B$? No

f) What is $n(\bar{B})$? $\bar{B} = \{1, 4, 6, 8, 9, 10\}$

$n(\bar{B}) = 6$

* Also $n(U) = 10$ and $n(B) = 4$

so, $n(\bar{B}) = n(U) - n(B) = 10 - 4 = 6$.

ex 5 Find the cardinal number for the following sets.

\# elements in a set

a) $\{1, 2, 4, 8, 16, 32, \dots, 1024\}$ $\boxed{11 \text{ elements}}$

$2^0 \quad 2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5 \quad \dots \quad 2^{10}$ (Guess + check)

b) $\{-10, -6, -2, 2, 6, \dots, 106\}$ arith $d = 4$

$a_n = a_1 + (n-1)d \longrightarrow 106 = -10 + (n-1)4$

$106 = -10 + 4n - 4$

$106 = -14 + 4n$

$120 = 4n$

$\boxed{30 = n}$

c) $\{x \mid x \in N, x < 0\}$

$\emptyset$

$n(\emptyset) = \boxed{0}$

note: N = natural #s = $\{1, 2, 3, 4, \dots\}$

How many subsets does the set $\{A, B, C, D\}$ have?

no elements	1 element	2 elements	3 elements	4 elem.
$\emptyset$	$\{A\}$	$\{AB\}$	$\{ABC\}$	$\{ABCD\}$
	$\{B\}$	$\{AC\}$	$\{ABD\}$	
	$\{C\}$	$\{AD\}$	$\{ACD\}$	
	$\{D\}$	$\{BC\}$	$\{BCD\}$	
		$\{BD\}$		
		$\{CD\}$		

$\boxed{16 \text{ subsets}}$

There is a shortcut! If a set has n elements, it will have 2^n subsets.

So, the last set had 4 elements and $2^4 = 16$ subsets.

How many subsets does $\emptyset$ have?

Just 1, itself! It has 0 elements, so using the 2^n we get $2^0 = 1$ subset.

end 2-2