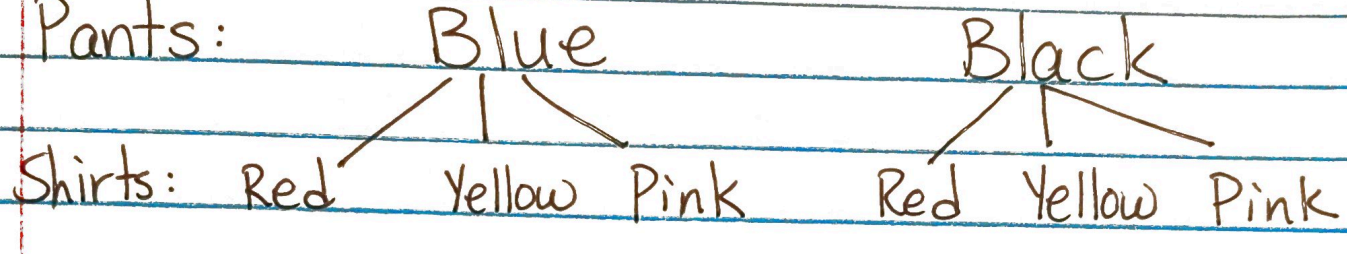
Section 9-4 Counting Techniques in Probabili

On a trip, you pack 2 pairs of pants and 3 shirts. How many outfits are possible?



There are 6 different outfits.

The Fundamental Counting Principle
The number of ways in which a series of successive things can occur is found by multiplying the number of ways in which each thing can occur.

In the above situation, the number of outfits could be found by:

$$2 \cdot 3 = 6$$

↑ #pants ↑ #shirts

ex 1 A discount paint manufacturer plans to make several different paints. The categories include:

6 Color: Red, Blue, White, Black, Green, Yellow
2 Type: Latex, Oil
3 Texture: Flat, Semigloss, High Gloss
2 Use: Indoor, Outdoor

How many different kinds of paint can be made?

$6 \cdot 2 \cdot 3 \cdot 2 = \boxed{72}$

ex 2 A keypad has buttons labeled 1-5 and requires a 3 digit code. How many codes are possible if:

a) digits can be repeated?

$\underset{1^{st}}{5} \cdot \underset{2^{nd}}{5} \cdot \underset{3^{rd}}{5} = \boxed{125}$

b) digits can not be repeated?

$\underset{1^{st}}{5} \cdot \underset{2^{nd}}{4} \cdot \underset{3^{rd}}{3} = \boxed{60}$

Before we get into other arrangement possibilities, let's look at factorials.

"5 factorial"

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$3! = 3 \cdot 2 \cdot 1 = 6$

$2! = 2 \cdot 1 = 2$

$1! = 1 = 1$

$0! = 1$ (Defn) (we'll see why later)

ex 3 Simplify

a) $\frac{9!}{3!}$ use calc or $\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 60,480$

b) $\frac{10!}{2!5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 15,120$

c) $\frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98 \cdots 3 \cdot 2 \cdot 1}{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1} = 9900$

A permutation is an ordered arrangement of items where:

1. no item is repeated and
2. the order is important

The number of permutations of n different objects using all of the objects is $n!$

ex 4 In how many different ways can the 12 football teams in the Big Ten Conference finish?

$12! = \boxed{479{,}001{,}600}$

ex 5 Look back at ex 4. Find the number of ways to have 1st and 2nd place finishers.

$\underset{1^{st}}{12} \cdot \underset{2^{nd}}{11} = \boxed{132}$

The permutations of n total objects taken r at a time is:

$$_nP_r = \frac{n!}{(n-r)!}$$

ex 6 In how many ways can a President, Vice President, and Treasurer be chosen from a class of 45 students?

$$_{45}P_3 = \underset{1^{st}}{45} \cdot \underset{2^{nd}}{44} \cdot \underset{3^{rd}}{43} = 85{,}140$$

syntax: 45 Math → PRB ↓ 2. nPr enter 3 enter

* Notice: $_nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ ← Defined to be 1

Permutations of Duplicate Items

The number of permutations of n items where p items are identical, q items are identical, r items are identical, etc is:

$$\frac{n!}{p!q!r!\ldots}$$

ex 7 In how many ways can we rearrange the letters in ILLINOIS (use all of the letters)?

n = 8 total
p = 3 I repeats
q = 2 L repeats

$$\frac{8!}{3!2!} = 3{,}360$$

A <u>combination</u> is an ordered arrangement of items where:
1. no item is repeated and
2. order is not important

The number of combinations of n total objects taken r at a time is:

$$nCr = \frac{n!}{(n-r)!\,r!} \quad \text{or} \quad \frac{nPr}{r!}$$

* Permutations = Order Matters
* Combinations = Order Does Not Matter

<u>ex 8</u> Label the following as either a permutation or combination.

a) Choosing 6 volunteers for medical research
Combination

b) An 8 character password
Permutation

c) The 3 digit combination to a combination lock
Permutation

d) The top 5 finishers of a race
Permutation

ex 9 On an exam, you must pick 2 essay questions from 6 essay questions. How many different ways are there to choose 2 questions?

order is not important

$_6C_2 = \boxed{15}$ (use calc)

ex 10 A committee consists of 5 men and 7 women. Three members are selected at random.

a) How many ways can this 3 member committee be all men?

$_5C_3 = \boxed{10}$

b) Find the probability that a 3 member committee selected at random will contain all men.

$$\frac{\#\text{ways to form a 3 man comm.}}{\#\text{ways to form a 3 person comm.}} = \frac{_5C_3}{_{12}C_3} = \frac{10}{220} = \boxed{\frac{1}{22}}$$

5 men 7 women

c) Find the probability that the 3 member committee contains 1 man and 2 women.

ways to form a 1 man and 2 w comm.
ways to form a 3 member comm.

$$= \frac{{}_5C_1 \cdot {}_7C_2}{{}_{12}C_3} = \frac{5 \cdot 21}{220} = \frac{105}{220} = \boxed{\frac{21}{44}}$$

d) Find the prob that the 3 member comm. contains 2 men and 1 woman.

$$\frac{{}_5C_2 \cdot {}_7C_1}{{}_{12}C_3} = \frac{10 \cdot 7}{220} = \frac{70}{220} = \boxed{\frac{7}{22}}$$

ex 11 A bag contains 25 batteries, 4 of which are dead thanks to my kid who changed the remote batteries and put the dead ones "back." Find the probability that if I pick 4 batteries from the bag at random:

a) that all 4 are dead

$$\frac{\#\text{ways to get 4 dead batteries by picking 4 at a time}}{\text{total \# ways to choose 4 batt from 25}}$$

$$= \frac{{}_4C_4}{{}_{25}C_4} = \boxed{\frac{1}{12{,}650}}$$

b) that at least 1 is dead

P(at least 1) = 1 − P(none) Formula

$$= 1 - \left(\frac{{}_{21}C_4}{{}_{25}C_4}\right)$$

$$= 1 - \frac{5985}{12{,}650} \approx \boxed{0.527}$$

end 9-4