Section 4-3 GCF and LCM

Every pair of numbers a and b have a common factor. The largest of the common factors is called the GCF or Greatest Common Factor (Divisor).

If $GCF(a, b) = 1$ then we say a and b are relatively prime.

Note: a and b do not have to be prime in order to be relatively prime. (ex: 4 and 9 are relatively prime)

We will look at 3 ways to find the GCF

1. intersection of sets (brute force)
2. prime factorization
3. Cuisenaire Rods (Colored Rods)

ex 1 Find the GCF. (same as GCD.)

a) 12 and 30 Let's use intersection of sets

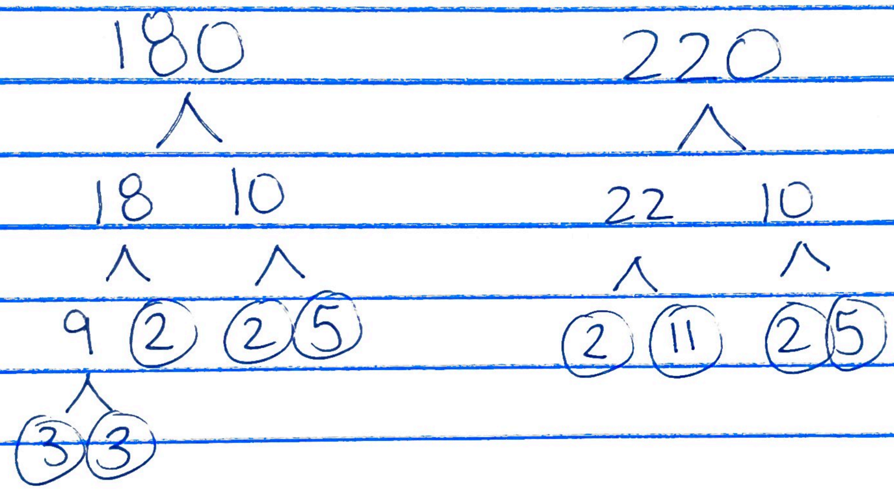
factors of 12: 1, 12, 2, 6, 3, 4
factors of 30: 1, 30, 2, 15, 3, 10, 5, 6

Common Factors are: 1, 2, 3, $\boxed{6 \leftarrow \text{GCF}}$

b) 180 and 220

Yikes! These probably have a lot of factors to list, so let's not use the intersection of sets.

Prime factorization



$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$220 = 2 \cdot 2 \cdot 5 \cdot 11$

$GCF = 2 \cdot 2 \cdot 5 = \boxed{20}$

c) 550 and 189

Prime factorization

550 → 55, 10
55 → 5, 11; 10 → 2, 5
5 11 2 5

189 → 9, 21
9 → 3, 3; 21 → 3, 7
3 3 3 7

$$\left.\begin{aligned} 550 &= 2\cdot 5\cdot 5\cdot 11 \\ 189 &= 3\cdot 3\cdot 3\cdot 7 \end{aligned}\right\} \text{No shared prime factors}$$

So, the GCF $(550, 189) = \boxed{1}$

* This means 550 and 189 are relatively prime.

d) 4 and 10 mathplayground.com

Let's use Cuisenaire Rods. $\boxed{2}$
(demo in video)

* GCF is the largest # that divides into both 4 & 10 with no remainder.

e) 4 and 8 $\boxed{4}$

Let's use Cuisenaire Rods again.
(in video)

f) 1 and 842

intersection of sets

factors of 1: 1
factors of 842: 1, 842, ... a bunch more

only common factor is the GCF

$\boxed{GCF = 1}$

Every number has an infinite number of multiples, so each pair has an infinite number of common multiples. The smallest of these common multiples is the LCM or Least Common Multiple.

We will look at 3 ways to find the LCM.

1. intersection of sets (brute force)
2. prime factorization
3. Cuisenaire Rods (Colored Rods)

ex 2 Find the LCM.

LCM = 6

a) 2 and 3

mult of 2: 2, 4, 6, 8, 10, 12, ...
mult of 3: 3, 6, 9, 12, ...

Use Cuisenaire Rods (see video)

b) 12 and 15 intersection of sets

mult of 12: 12, 24, 36, 48, 60, 72, 84, 96, ...
mult of 15: 15, 30, 45, 60, 75, ...

$\boxed{\text{LCM is } 60}$

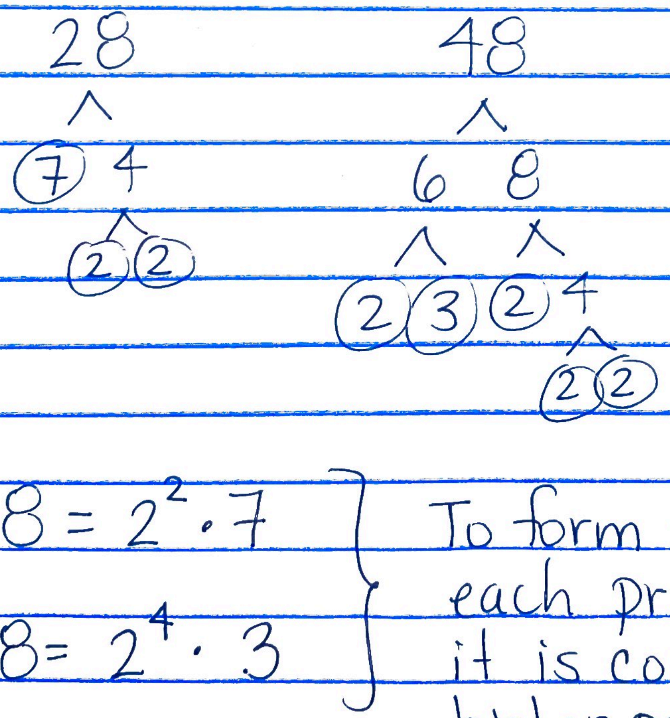
c) 1 and 842

mult of 1: 1, 2, 3, 4, 5, 6, ..., 842, 843, ...
mult of 842: 842, ...

$\boxed{LCM = 842}$

d) 28 and 48

Prime factorization



(exponent form!!)

$28 = 2^2 \cdot 7$

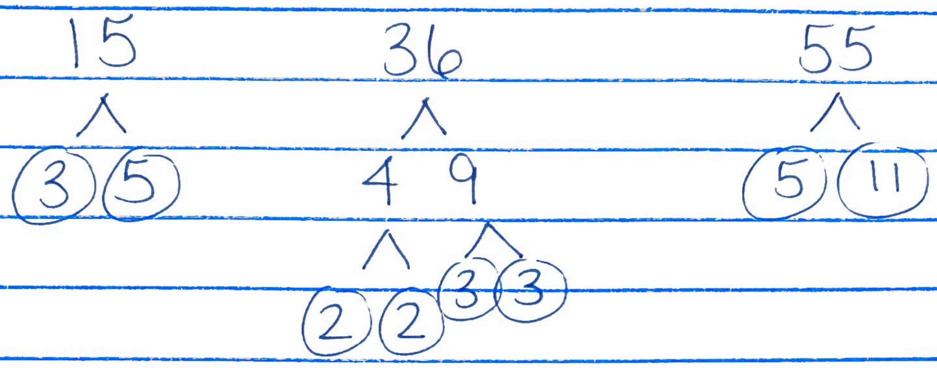
$48 = 2^4 \cdot 3$

To form the LCM, use each prime factor + if it is common, choose the higher power.

$LCM = 2^4 \cdot 3 \cdot 7 = \boxed{336}$

e) 15, 36, and 55

Prime factorization



$15 = 3 \cdot 5$
$36 = 2^2 \cdot 3^2$
$55 = 5 \cdot 11$

$LCM = 3^2 \cdot 2^2 \cdot 5 \cdot 11 = \boxed{1980}$

choose
b/w 3 and 3^2
so choose the
higher power

exponent
form

end 4-3