Section 6-1 Rational Numbers

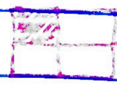
Defn A rational number is a number that can be written as p/q where $q \neq 0$ and $p, q \in \boxed{\text{integers}}$.

Defn A fraction is a number that can be written as p/q where $q \neq 0$ and $p, q \in \boxed{\text{real numbers}}$.

* The rational numbers are a subset of the set of fractions.

* All rational #s are fractions, but not all fractions are rational #s.
ex: $\frac{\sqrt{2}}{3}$ is a fraction, but not a rational #

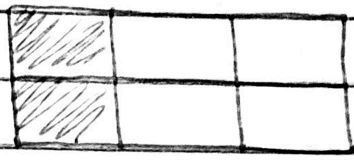
We can utilize fractions in 4 ways:

1. Division ex: Solve $2x = 1$
2. Part-to-Whole ex: What frac is shaded? 
3. Ratio ex: The ratio of men to women is 1:4.
4. Probability ex: The prob of rolling a 4 is $1/6$.

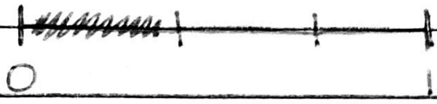
* The rational numbers are dense. This means there is no "next" rational number.

Illustrations of Fractions

1. Area model



2. Number line



3. Set model

* Each one of these illustrates $1/3$.

Fractions are subdivided into proper and improper fractions.

* A fraction a/b is:
  proper if $|a| < |b|$ and
  improper if $|a| \geq |b|$.

Fundamental Theorem of Fractions:
(Equality of Fractions)

one

$$\frac{a}{b} = \frac{a\boxed{k}}{b\boxed{k}}, \quad b \neq 0, \; k \neq 0$$

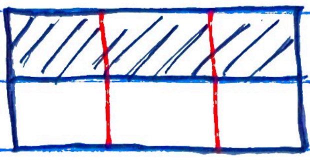
ex.: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10} \ldots$

completely reduced

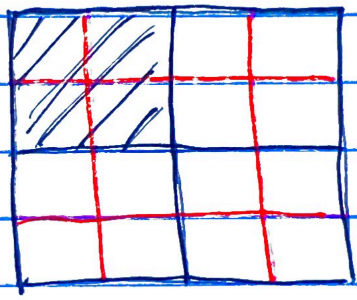
A fraction a/b is in simplest form if a and b are relatively prime.
[Recall relatively prime $\Rightarrow$ $GCF(a,b)=1$]

ex 1 Use an area model to illustrate:

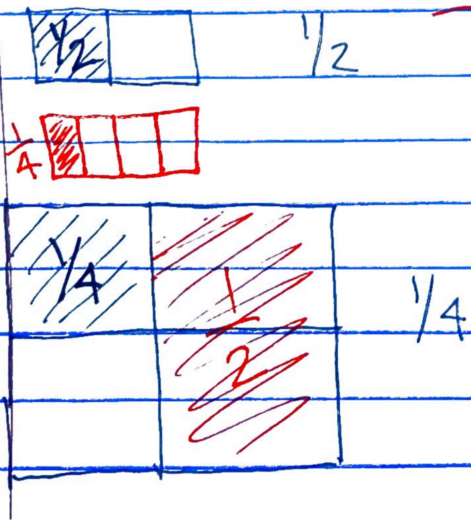
a) $\frac{1}{2} = \frac{3}{6}$



b) $\frac{1}{4} = \frac{4}{16}$



ex 2 A 4^{th} grader says $1/4$ is larger than $1/2$ and draws this as "proof." How do you respond?



The wholes are not the same, so you can't compare.

To compare 2 fractions you can:

1. get a common denominator, then compare the numerators. $3/10$ $4/10$

2. get a common numerator, then compare the denominators $1/5$ $1/6$

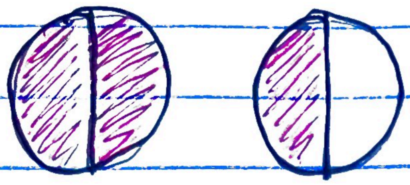
3. use the property that $a/b = c/d$ if and only if $ad = bc$. [This property extends to < and > too.]

Why? $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{\boxed{ad}}{bd} = \frac{\boxed{cb}}{bd}$

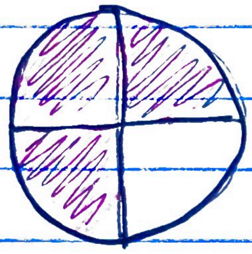
4. use fraction bars (shown in video)

5. use area models

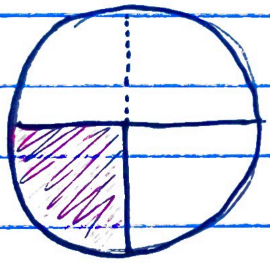
ex 3 What fraction is shaded?

a) 

$1\frac{1}{2}$ or $\frac{3}{2}$

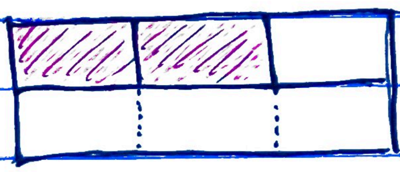
b) 

$\frac{3}{4}$

c) 

$\frac{1}{4}$

The pieces need to be equal sized.

d) 

$\frac{2}{6}$

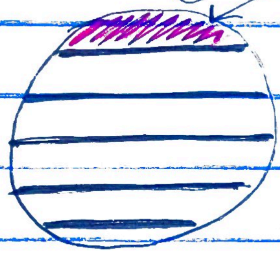
e) 

The line from 0 to 1 is cut into 8 equal segments.

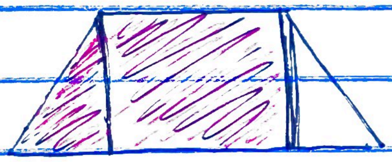
$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

$\frac{3}{8}$

ex 4 Explain the problems with the shadings below.

a) 

$\frac{1}{6}$

b) 

$2/3$

These are not cut into equal sized pieces.

end 6-1