Section 6-2 Addition + Subtraction of $\mathbb{Q}$

Properties of Rational Number Addition

1. closure property
The sum of any 2 rational numbers is always a rational number.

2. commutative
ex: $\frac{2}{3} + \frac{1}{6} = \frac{1}{6} + \frac{2}{3}$

3. associative
ex: $\frac{3}{7} + \left(\frac{1}{3} + \frac{2}{3}\right) = \left(\frac{3}{7} + \frac{1}{3}\right) + \frac{2}{3}$

4. Additive Identity Property
ex: $\frac{2}{5} + 0 = \frac{2}{5}$ or $0 + \frac{2}{5} = \frac{2}{5}$

* 0 is the additive identity element

5. Additive Inverse Property
ex: $\frac{2}{7} + \left(\frac{-2}{7}\right) = 0$

* $-a$ is the additive inverse of a

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

ex: $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

* Think of the denominators as units.
1 ft + 2 inches ≠ 3 ft inches
You have to get the same unit in order to add or subtract.

Similarly $\frac{1}{5} + \frac{2}{3} \neq \frac{3}{8}$

In order to add or subtract fractions, you must have a common denominator.

ex 1 Add $1/5 + 2/3$

LCM(5, 3) = 15

$$\left(\frac{1}{5} \cdot \frac{3}{3}\right) + \left(\frac{2}{3} \cdot \frac{5}{5}\right)$$

$$= \frac{3}{15} + \frac{10}{15} = \frac{13}{15}$$

$$\frac{1}{5} + \frac{2}{3} = \frac{3 \cdot 1 + 5 \cdot 2}{5 \cdot 3}$$

$$= \frac{3 + 10}{15}$$

$$= 13/15$$

ex 2 Use fraction bars to add $\frac{1}{3} + \frac{1}{2}$

(did in video)

$\frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$

ex 3 Subtract $\frac{5}{8} - \frac{1}{4}$ then model with fraction bars.

LCM (8, 4) = 8

$\frac{5}{8} - \left(\frac{1}{4} \cdot \frac{2}{2}\right)$

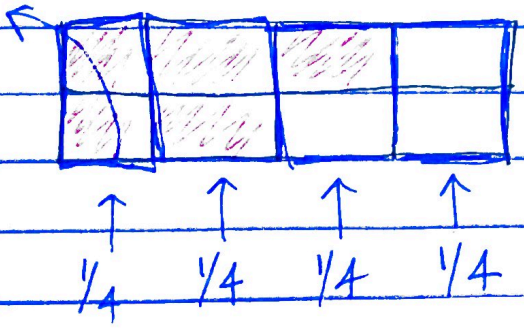
$= \frac{5}{8} - \frac{2}{8}$

$= \frac{3}{8}$

$\frac{5}{8} - \frac{1}{4} = \frac{20-8}{32}$

$= \frac{12 \div 4}{32 \div 4}$

$= \frac{3}{8}$



Mixed numbers are made up of an integer and a fractional part of another integer.

ex 4 Convert $6\frac{2}{7}$ to a fraction

$$6\frac{2}{7} = \frac{6}{1} + \frac{2}{7} = \left(\frac{6}{1}\cdot\frac{7}{7}\right) + \frac{2}{7} = \frac{42}{7} + \frac{2}{7} = \boxed{\frac{44}{7}}$$

Shortcut:

$$6\frac{2}{7} = \frac{7\times 6+2}{7} = \frac{44}{7}$$

ex 5 Convert $-13/5$ to a mixed number.

$$-\frac{13}{5} \div$$

$$5\overline{)13}\quad 2 \quad (-10) \quad \text{rem } 3$$

$$-2\,\tfrac{3}{5}$$

2 = Quotient

end 6-2

ex 6 Add $4\frac{2}{5} + 2\frac{1}{2} + \frac{3}{5}$

$4 + \frac{2}{5} + 2 + \frac{1}{2} + \frac{3}{5}$ compatible numbers

$6 + \frac{5}{5} + \frac{1}{2}$

$6 + 1 + \frac{1}{2} = 7\frac{1}{2}$

A good way to estimate fractions is to use benchmark fractions like $0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$

ex 7 Estimate $\frac{4}{6} + \frac{7}{8}$

$\frac{1}{2} + 1 \approx 1\frac{1}{2}$

* fracs of the form $\frac{n-1}{n}$ are easily recognized by kids as being close to 1.

* $\frac{4}{6}$ is not as easily recognized. You have to think about half of 6 & how it compares to 4.

6-2 Add/Sub Mixed Numbers (Extra Stuff)

Addition

There are 2 ways to add or subtract mixed numbers.

1. vertically / regrouping
2. change to improper fractions

ex 1 Add $1\frac{3}{4} + 5\frac{2}{3}$

vertical / regrouping

$$\begin{array}{r} 1\ 3/4 \\ +\ 5\ 2/3 \end{array}$$

need a common denom.

$$\begin{array}{r} 1\ 3/4 \cdot 3/3 \\ +\ 5\ 2/3 \cdot 4/4 \end{array}$$

$$\Rightarrow \begin{array}{r} 1\ 9/12 \\ +\ 5\ 8/12 \\ \hline 6\ 17/12 \end{array}$$

we have to change this to a mixed #

$\frac{17}{12} \Rightarrow 12\overline{)17} \Rightarrow 1\frac{5}{12}$ (1 remainder 5)

So, $6\frac{17}{12} = 6 + \frac{17}{12} = \underbrace{6 + 1}_{\text{This is the regrouping I mentioned.}} + \frac{5}{12} = \boxed{7\frac{5}{12}}$

change to improper fractions

$1\frac{3}{4} + 5\frac{2}{3}$

$= \frac{7}{4} + \frac{17}{3}$ get a common denom

$= \left(\frac{7}{4} \cdot \frac{3}{3}\right) + \left(\frac{17}{3} \cdot \frac{4}{4}\right)$

$= \frac{21}{12} + \frac{68}{12} = \frac{89}{12}$ Now change to a mixed #

$$\begin{array}{r} 7 \\ 12\overline{)89} \\ -84 \\ \hline 5 \end{array}$$

$\boxed{7\frac{5}{12}}$

ex 2 Subtract $4\frac{1}{5} - 2\frac{7}{10}$

vertical / regrouping

$$\begin{array}{r} 4\frac{1}{5} \\ -\,2\frac{7}{10} \end{array} \xrightarrow{\text{get a common denom}} \begin{array}{r} 4\frac{1}{5} \cdot \frac{2}{2} \\ -\,2\frac{7}{10} \end{array} \Rightarrow \begin{array}{r} 4\frac{2}{10} \\ -\,2\frac{7}{10} \end{array}$$

You can't subtract 7 from 2 so you must "borrow" or regroup

$$\begin{array}{r} 3 \\ 4\frac{2}{10} + \frac{10}{10} \\ -\,2\frac{7}{10} \end{array} \Rightarrow \begin{array}{r} 3\frac{12}{10} \\ -\,2\frac{7}{10} \\ 1\frac{5}{10} \end{array} = \boxed{1\frac{1}{2}}$$

We can eliminate the need for "borrowing" if we change the mixed #s to improper fractions.

Change to improper fractions

$4\frac{1}{5} - 2\frac{7}{10}$

$= \frac{21}{5} - \frac{27}{10}$ get a common denominator

$= \frac{21}{5} \cdot \frac{2}{2} - \frac{27}{10}$

$= \frac{42}{10} - \frac{27}{10} = \frac{15}{10} \div \frac{5}{5} = \frac{3}{2} = \boxed{1\frac{1}{2}}$

I didn't show the long division here bc I knew you recognized $\frac{3}{2}$ as $1\frac{1}{2}$.