

Section 3.4 Logical Arguments Pg 17

An argument consists of 2 parts:

1. the given statements called premises
2. the conclusion that is based on these premises

ex 1 Use a truth table to determine if the argument is valid or invalid.

premises { If it is cold, then my car will not start.
My car starts.
∴ It is not cold.

symbol for therefore

conclusion

Symbolically

p: It is cold.

q: My car starts.

$$\begin{array}{l} p \rightarrow \sim q \\ q \\ \hline \therefore \sim p \end{array}$$
 } premises
conclusion

An argument is valid if the conjunction of the premises implies the conclusion is a tautology.

We need a truth table for:

$$((p \rightarrow \sim q) \wedge q) \rightarrow \sim p$$

premise premise conclusion

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p	q	$\sim q$	$p \rightarrow \sim q$	$(p \rightarrow \sim q) \wedge q$	$\sim p$	$((p \rightarrow \sim q) \wedge q) \rightarrow \sim p$
T	T	F	F	F	F	T
T	F	T	T	F	F	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T

all T
So the argument is valid!

Look at the last column - if it is a tautology (all Ts) then the argument is valid.

If there is one or more Fs in the last column, the argument is invalid.

ex 2 Use a truth table to determine if the following argument is valid or invalid.

$$\begin{array}{l} p \rightarrow q \\ p \wedge q \\ \hline \therefore p \end{array} \quad \left. \begin{array}{l} \text{premises} \\ \text{conclusion} \end{array} \right\}$$

Part 1: Write the argument as an implication.
form: (premise $\wedge$ premise) $\rightarrow$ conclusion
 $((p \rightarrow q) \wedge (p \wedge q)) \rightarrow p$

Parts 2+3: Draw a truth table.

	p	q	$p \rightarrow q$	$p \wedge q$	$(p \rightarrow q) \wedge (p \wedge q)$	$\star \rightarrow p$
Part 2	T	T	T	T	T	T
	T	F	F	F	F	T
Part 3	F	T	T	F	F	T
	F	F	T	F	F	T

copied from summary sheet

Part 4 Is the argument valid or invalid?

valid bc last column is all T

ex 3 Use a truth table to determine if the argument is valid or invalid.

$$\begin{array}{c} p \leftrightarrow q \\ q \\ \hline \therefore \sim p \end{array}$$

Part 1: Write the argument as an implication.
 $((p \leftrightarrow q) \wedge q) \rightarrow \sim p$

Parts 2 + 3: Draw a truth table.

	p	q	$p \leftrightarrow q$	$(p \leftrightarrow q) \wedge q$	$\sim p$	$\star \rightarrow \sim p$
Part 2	T	T	T	T	F	F
	T	F	F	F	F	T
Part 3	F	T	F	F	T	T
	F	F	T	F	T	T

↑ summary sheet

invalid

Part 4: Is the argument valid or invalid?

invalid bc there is at least one F in the last column

Some arguments are common enough to have names. If we run across these, we can skip building a truth table.

Valid Argument Forms

1. Law of Detachment

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

2. Law of Contraposition

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

3. Syllogism

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

4. Law of Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

or

$$\begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

Invalid Argument Forms

1. Fallacy of the Converse

$$\begin{array}{l} p \rightarrow q \\ q \\ \hline \therefore p \end{array}$$

2. Fallacy of the Inverse

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

3. Fallacy of the Inclusive Or

$$\begin{array}{l} p \vee q \\ p \\ \hline \therefore \sim q \end{array}$$

ex 4 (from HW #6 - #10)

Write the argument in symbols, then decide whether the argument is valid by using the common forms of valid arguments and fallacies.

If it is sunny, I will wear SPF 50 sun block.
 It is not sunny.
 $\therefore$ I will not wear SPF 50 Sun block.

p : It is sunny.

q : I will wear SPF 50 sun block.

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

Fallacy of Inverse
Invalid