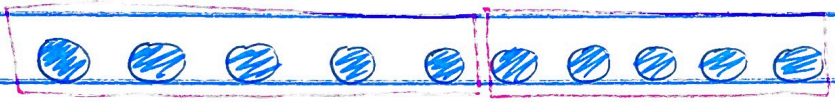
Section 3-5 Division of Whole #s

Division is modeled in 3 ways:

1. Set (Partitive) Model

In this model, we split the total into equal sized groups. The quotient (answer) comes from "How many are in each group?" (set)



$10 \div 2 = 5$

- 10 ↑ total
- 2 ↑ split into 2 equal groups
- 5 ↖ 5 in each group

10 cookies split equally between 2 friends. How many cookies does ~~each friend~~ get?

2. Repeated Subtraction Model

In this model, we repeatedly subtract off equal sized groups. The quotient comes from "how many groups?"

$10 \div 2 = 5$

- 10 ↑ total
- 2 ↑ rep sub 2 until nothing is left
- 5 ↖ How many groups get subtracted?

10 cookies sell cookies in 2 cookie bundles. How many bundles?

3. Missing Factor

In this model, we rewrite the division problem as its related multiplication problem.

$10 \div 2 = \square \Rightarrow \square \times 2 = 10$

dividend, divisor, quotient

This is helpful when transitioning from multiplication to division.

Division Algorithm

Given any whole numbers a and b with $b \neq 0$, there exist unique whole numbers q (quotient) and r (remainder) such that:

$$a = bq + r \text{ with } 0 \leq r < b$$

$$\begin{array}{r} 1 \\ 5\overline{)7} \\ -5 \\ \hline 2 \end{array}$$

$5(1) + 2 = 7$

The remainder is always smaller than the divisor but larger or equal to zero.

Properties of Whole # Division.

The whole #s for division are:

1. not closed
   ex: $2 \div 5$ is not a whole #

2. not commutative
   ex: $2 \div 5 \neq 5 \div 2$

3. not associative
   ex: $10 \div (5 \div 2) \neq (10 \div 5) \div 2$

4. do not have an identity element
   ex: $10 \div 1 \neq 1 \div 10$

Division with Zero

The division problems $4 \div 0$, $0 \div 4$, and $0 \div 0$ are often misunderstood. Let's look at them closer.

Recall: $10 \div 2 = 5 \Rightarrow 5 \cdot 2 = 10$

1. $4 \div 0 = \square \Rightarrow \square \cdot 0 = 4$

undefined or no solution

There is no # that when mult by 0 gives you 4.

2. $0 \div 4 = \square \Rightarrow \square \cdot 4 = 0$ → $\boxed{0}$

The answer is 0 since $0 \cdot 4 = 0$.

3. $0 \div 0 = \square \Rightarrow \square \cdot 0 = 0$ → indeterminate

Any # will work here. So there is no unique solution. We say this is an indeterminate form since no unique solution can be determined.

Before we move on to algorithms, let's look at some problems and classify them.

* See problem classification sheets *
(separate pages)

Division of Whole Numbers

Discuss whether the following should be modeled using the partitive approach or the repeated subtraction approach.

1. A company shipped 192,600 bushels of wheat in 9 railroad cars. Find the amount of wheat shipped in each car.

Partitive split 192,600 into 9 equal groups

2. A chef purchased a roast that weighed 10 pounds. How many 2 pound servings can be cut from this roast?

Rep Sub Rep taking away 2 from 10

3. A retail outlet can store 270 tires equally among 15 shelves. How many tires can be stored on one shelf?

Partitive split 270 into 15 groups

4. Individual cereal boxes contain 2 ounces of cereal. How many individual boxes can be filled with 600 ounces of cereal?

Rep Sub Rep taking away 2 from 600

5. Carissa is building bookcases that are 3 feet wide. How many complete shelves can be cut from a 15 foot long board?

Rep Sub $15-\underbrace{3-3-3-3-3}_{5\text{ times}}=0$

6. Three charities worked together to raise $548,000. If the money is split equally between the charities, how much will each charity get?

Partitive split 548,000 into 3 grps

7. A bag of rice contains 18 ounces of rice. How many 3 ounce portions can be served from this bag?

Rep Sub $18-\underbrace{3-3-3-3-3-3}_{6\text{ times}}=0$

8. A car costs $9,216 and has to be paid off in 48 equal monthly payments. How much is the monthly payment?

Partitive 9216 split into 48 grps

Division Algorithms

1. Standard Algorithm

$135 \div 5$

$$
\begin{array}{r|l}
 & 27 \\
\hline
5 & 135 \\
 & -10\downarrow \\
\hline
 & 35 \\
 & -35 \\
\hline
 & 0
\end{array}
$$

Divide
Mult
Sub
Bring Down

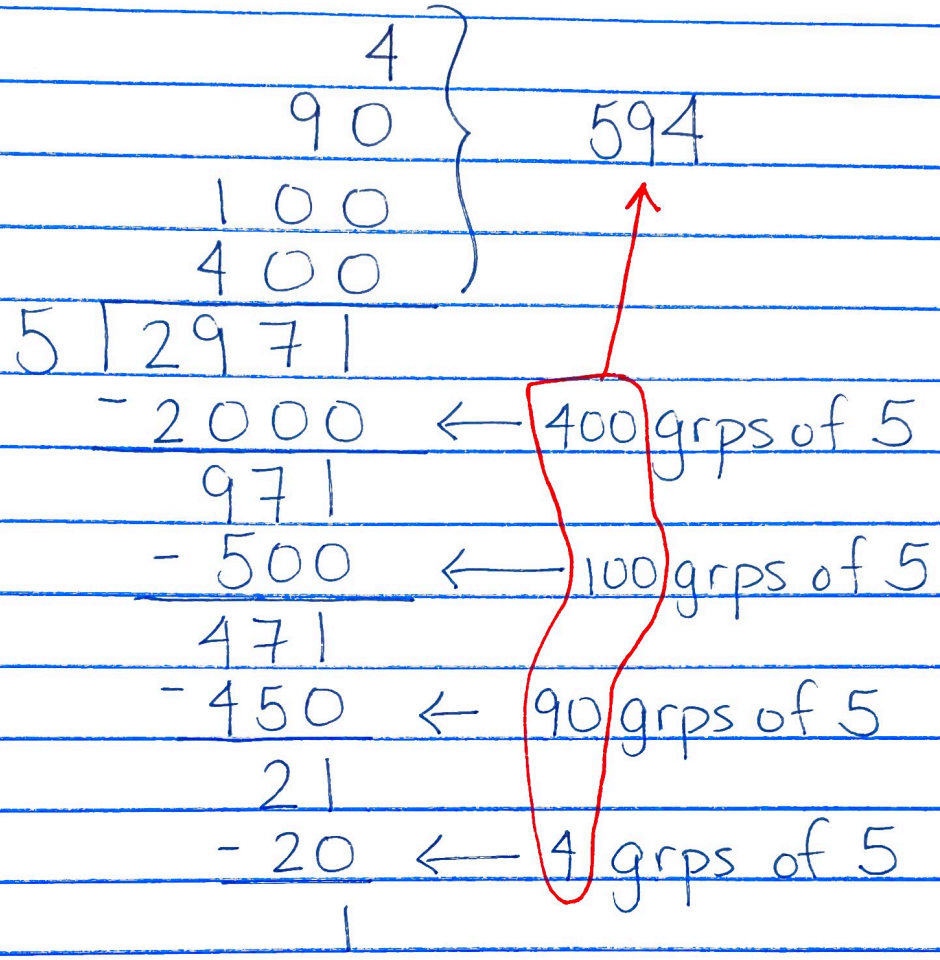
* it is very important that you line up the place values *

2. Partial Quotients (Scaffolding / Rep. Sub)

$$
\begin{array}{r|l}
 & 7 \\
 & 20 \\
\hline
5 & 135 \\
 & -100 \quad \leftarrow 20 \text{ grps of } 5 \\
\hline
 & 35 \\
 & -35 \quad \leftarrow 7 \text{ grps of } 5 \\
\hline
 & 0
\end{array}
$$

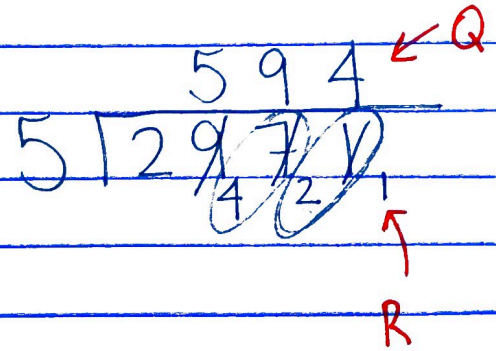
7 } total of 27
20 }

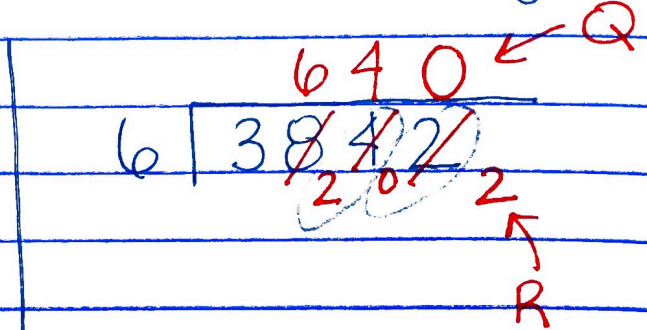
Another ex of Scaffolding:



3. Short Division

This algorithm is the standard algorithm, but with subtraction done mentally.



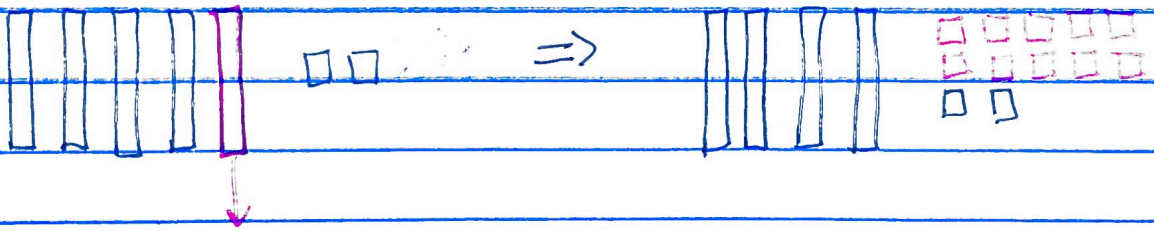


4. Base 10 Blocks

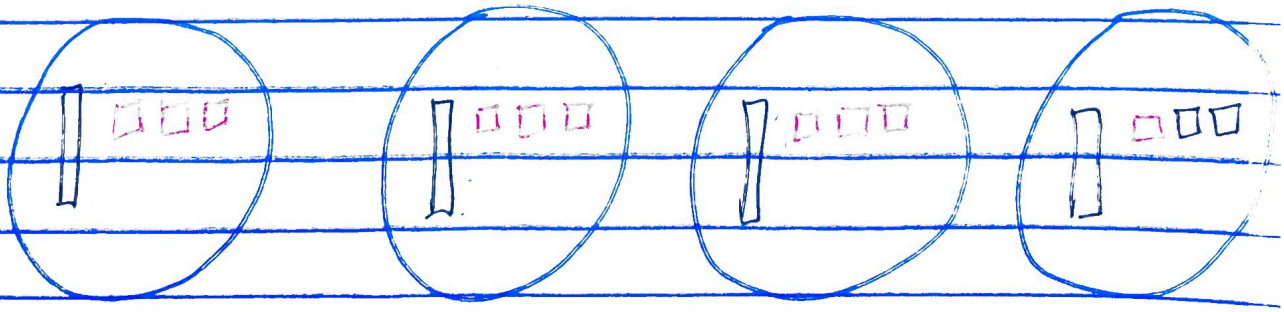
$52 \div 4 = 13$

We start with 52 (5 longs + 2 units). We can either split 52 into 4 equal sized groups (partitive) or we can repeatedly subtract 4 from 52 until nothing is left.

Let's use a partitive approach.



exchange for 10 units



There is 1 long + 3 units in each group, so the quotient is 13.

Division with Other Bases

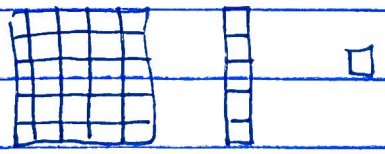
This is way more challenging than add, sub, or mult in other bases. We will only look at division with other bases using base blocks.

* If the divisor is a single digit, use partitive.
* If the quotient is a single digit, use repeated subtraction.

ex1 Model the following using base blocks.

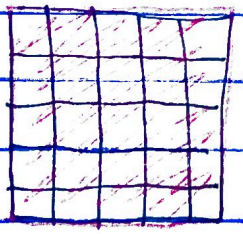
a) $103_{five} \div 2_{five} = 24_{five}$

Base 5 blocks:



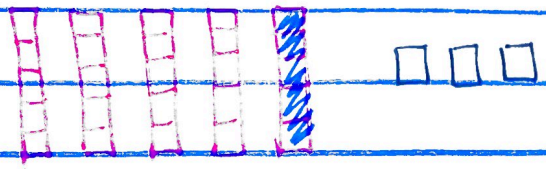
25 5 1

Since the divisor is a single digit, we will split 103_{five} into 2 equal grps (partitive).

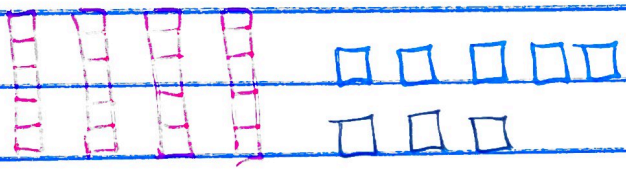


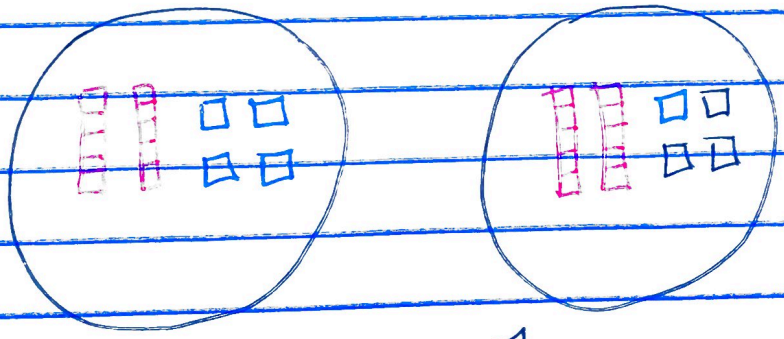
← 103_{five}

exchange the 1 flat (25) for 5 longs.



I still can't split this into 2 equal groups, so let's exchange 1 long for 5 units.



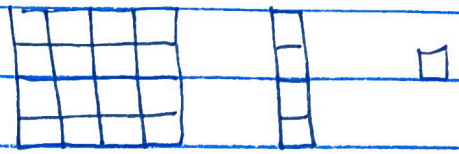


2 equal groups each containing 2 longs + 4 units.

So the quotient is 24 five.

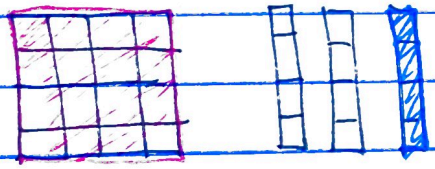
b) $130_{four} \div 32_{four} = 2_{four}$

Base 4 blocks:



16 4 1

Since the quotient is a single digit, we will start with 130_{four} and repeatedly subtract 32_{four} until nothing is left.



1 3 0

I want to subtract 32_{four} (3 longs + 2 units) so I need to exchange 1 flat for 4 longs and 1 long for 4 units.

32_{four} 32_{four}

32_{four} was rep. subtracted 2 times, so the quotient is 2.

Mental Division Techniques

1. Compatible Numbers

ex 2 Divide mentally

a) $3\overline{)75} \Rightarrow 3\overline{)\overset{20+5}{60+15}} = 25$

b) $8\overline{)232} \Rightarrow 8\overline{)\overset{30-1}{240-8}} = 29$

2. Breaking Up the Dividend

ex 3 Divide mentally

a) $4\overline{)324} \Rightarrow 4\overline{)\overset{81}{32}4}$

b) $25\overline{)5075} \Rightarrow 25\overline{)\overset{203}{50}75}$ need a 0 Place holde

* This technique is similar to compatible #s.

$25\overline{)5075} \Rightarrow 25\overset{200+3}{\overline{)5000+75}}$

end 3-5