Math 206 Section 9-1 Determining Probabilities

The study of probability came from a desire to understand games of chance like those involving cards or dice. Probability has since become useful in insurance, marketing, polling, investment decision-making, and testing experimental drugs.

* Only grade 7 has specific common core standards for probability.

Defn Probabilities are ratios, expressed as fractions, decimals, or percents that are found by looking at outcomes of an experiment.

An experiment is an activity where the results are observed and recorded.

An outcome is the result from a single trial of an experiment.

The set of all of the outcomes in an experiment is the sample space.

Any subset of the sample space is an event.

ex 1 Determine the sample space.

a) rolling a 6 sided fair die
{1, 2, 3, 4, 5, 6}

b) flipping a fair coin
{H, T}

c) rolling an 8 sided fair die
{1, 2, 3, 4, 5, 6, 7, 8}

d) flipping 2 fair coins simultaneously
{HH, HT, TH, TT}

e) Choosing a marble from a bag that contains 3 red marbles and 4 blue marbles.
{R, R, R, B, B, B, B} or {Red, Blue}

ex 2 List an event associated with each part of ex 1.

a) {2, 4, 6} or {2, 3, 5} or {4} or {1, 6} ...
b) {H} or {T}
c) {1, 2, 3, 4, 5, 6, 7} or {1, 3, 5, 7} ...
d) {HH, TT}
e) {Red} or {Blue} or {Red, Blue}

Note: An outcome (result) is a single outcome whereas an event can be a single or multiple outcomes. (results)

We will discuss 2 types of probability:

1. Experimental (empirical) probabilities are found by performing an experiment and recording the outcomes of that experiment.

2. Theoretical (classical) probabilities are the ideal probabilities assigned to an experiment that disregard physical or material imperfections.

* In this chapter we use the word "probability" to mean theoretical probability.

There are instances when we would rather have actual data than a theoretical probability.

For instance, actual data shows that the prob of you being killed:

in a plane crash = 1/1 million
by lightning = 1/3 million
by a shark = 1/100 million
in a car accident = 1/7,000
from a heart attack = 1/400
by a cow = 1/5 million

There are people who work for insurance companies whose job it is to calculate these probabilities. They are called Actuaries and they make sweet $

Back to theoretical Probabilities...

$$\underbrace{P(E)}_{\text{Prob of an event}} = \frac{n(E)}{n(S)} = \frac{\#\text{ outcomes in event}}{\#\text{ outcomes in sample space}}$$

We can also write this as:

$$P(E) = P(e_1) + P(e_2) + \cdots + P(e_K)$$
where $E = \{e_1, e_2, \ldots, e_K\}$

ex 3 You roll a fair 6 sided die. Find the probability of rolling:

a) 1 or 6 $\quad \frac{n(E)}{n(S)} = \frac{2}{6} = \boxed{\frac{1}{3}}$

b) a number less than 5 $\rightarrow 1, 2, 3, 4 \quad \frac{4}{6} = \boxed{\frac{2}{3}}$

c) a number 5 or less $\rightarrow 1, 2, 3, 4, 5 \quad \boxed{\frac{5}{6}}$

Sum	2	3	4	5	6	7	8	9	10	11	12
prob	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

36 outcomes

ex 4 You roll a pair of fair 6 sided dice. Find the probability that the sum of the numbers facing up is:

a) 5 1,4 , 4,1 , 2,3 , 3,2 $4/36 = \boxed{1/9}$

b) larger than 7 $\frac{5+4+3+2+1}{36} = \frac{15}{36} = \boxed{\frac{5}{12}}$

~~8: 2,6 , 6,2 , 4,4 , 3,5 , 5,3~~

c) prime 2, 3, 5, 7, 11 ← sum!

$1+2+4+6+2/36 = 15/36 = \boxed{5/12}$

d) 15 $\frac{0}{36} = \boxed{0}$

Some things to know about probabilities:

1. prob is never negative
2. prob is always between 0 and 1 (can be 0 or 1, but never greater than 1)
3. prob of 0 = impossible event
4. prob of 1 = certain event
5. Sum of prob for every outcome in a sample space is 1. ($P(S) = 1$)
6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

* Two events are mutually exclusive if they have no elements in common. (i.e., $A \cap B = \emptyset$)

half red + half black
4 suits: clubs, spades, diamonds, hearts
Each suit has 13 cards: A | 2, 3, 4, 5, 6, 7, 8, 9, 10 | J, Q, K
number cards — face cards

ex 5 Using a standard 52 card deck, find the prob of drawing:

a) a 6 $4/52 = \boxed{1/13}$

b) a red card $26/52 = \boxed{1/2}$

c) a red 6 $2/52 = \boxed{1/26}$

d) a club $13/52 = \boxed{1/4}$

e) a face card $12/52 = \boxed{3/13}$

f) a 2 **or** heart (**or** → ∪, **and** → ∩)

$P(2 \cup H) = P(2) + P(H) - P(2 \cap H) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$

g) the 2 of hearts (**and**)

$P(2 \cap H) = \boxed{1/52}$

h) a red **or** black card

$P(R \cup B) = P(R) + P(B) - \cancel{P(R \cap B)} = 1/2 + 1/2 = \boxed{1}$

i) a number card that is even

5 # cards that are even per suit

$5 \cdot 4 = 20$ total even # cards

$$\frac{20}{52} = \boxed{\frac{5}{13}}$$

ex 6 A school raffle sells 125 tickets and 10 of those are winners. What is the prob of winning?

$$\frac{10}{125} = \boxed{\frac{2}{25}}$$

ex 7 A cooler has 10 Orange Fantas and 4 Strawberry Fantas. What is the prob of reaching in blindly and getting a strawberry Fanta?

$$\frac{4 \leftarrow \#\text{ strawberry}}{14 \leftarrow \text{total }\#} = \boxed{\frac{2}{7}}$$

ex 8 There are 36 outcomes in the sample space for rolling 2 fair 6 sided dice. What is the prob of NOT rolling "snake eyes" (2 ones)?

only 1 way to get snake eyes
SO, there are 35 ways to NOT get snake eyes.

$$\boxed{35/36}$$

2 dice	P(snake eyes) = 1/36	P(not snake eyes) = 35/36
coin	P(heads) = 1/2	P(tails) = 1/2
1 die	P(4 or less) = 4/6	P(greater than 4) = 2/6

These are examples of complementary events.

Defn Two events whose union is the sample space and whose intersection is empty are called complementary.

* We use $\bar{A}$ to denote the complement of A.
* $A \cup \bar{A} = S$
* $A \cap \bar{A} = \emptyset$
* $P(A) + P(\bar{A}) = P(S) = 1$

Be careful!!

If 2 events are complementary then they are always mutually exclusive.

If 2 events are mutually exclusive they may or may not be complementary.
(ex: rolling a # less than 3 and rolling a # 6)

Since $P(A) + P(\bar{A}) = 1$ we get the variation:

$$P(\bar{A}) = 1 - P(A)$$

* Recall from sets back in Chapter 2 that we typically use the word "not" with complements.

ex 9 What is the probability of not drawing a queen from a standard 52 card deck?

$P(\bar{Q}) = 1 - P(Q) = 1 - 4/52 = 48/52 = 12/13$

ex 10 Describe an event for the following.

Probability	Event (Answers will vary.)
1	flipping a fair coin & getting H or T
0.999	dreary weather in Feb.
0.5	flipping a coin & getting "H"
0.0001	getting killed by a cow
0	flipping a coin & getting shark

ex11 The probability of a coin landing on tails is 1. Explain how this can happen.

The coin is not fair. If the coin has tails on both sides then $P(T) = 1$.

End 9-1.