Section 4-2 Prime + Composite Numbers

We can classify whole numbers according to their factors. Most numbers have factors that come in pairs of distinct numbers. The numbers who do not are the perfect squares.

Why?

		# factors is
Factors of 10:	1, 10, 2, 5	even
Factors of 12:	1, 12, 2, 6, 3, 4	even
Factors of $\boxed{16}$:	1, 16, 2, 8, 4, 4	odd
Factors of 20:	1, 20, 2, 10, 4, 5	even
Factors of $\boxed{25}$:	1, 25, 5, 5	odd

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, ...

A positive integer with exactly 2 distinct factors: 1 and itself is prime.
A positive integer with 3 or more factors is composite.

* The number 1 has only 1 factor, thus it is the only positive integer that is neither prime nor composite.

Primes: 2, 3, 5, 7, 11, 13, 17, 19, ...

ex 1 Is 53 prime or composite?

Is it divisible by 2? 3? 4? 5? 6? ...

There has to be an easier way!

Prime Number Test

If there is no prime number less than $\sqrt{n}$ (rounded up) that is a factor of n, then n is a prime number.

Back to ex 1: $\sqrt{53}$ rounded up = 8

Test primes: 2, 3, 5, 7

Is 53 divisible by 2? 3? 5? 7?

No No No No

$\boxed{\text{So, 53 is prime.}}$

ex 2 Determine if the following are prime or composite.

a) 401 $\sqrt{401} \xrightarrow[\text{up}]{\text{round}} 21$

$\boxed{\text{prime}}$

Test Primes: $\cancel{2}, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{11}, \cancel{13}, \cancel{17}, \cancel{19}$

401 ÷ 7
is not whole #

b) 221 $\sqrt{221} \xrightarrow{\text{roundup}} 15$

$\boxed{\text{Composite}}$

Test Primes: $\cancel{2}, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{11}, 13$

$221 \div 13 = 17$

c) 667 $\sqrt{667} \xrightarrow{\text{roundup}} 26$

$\boxed{\text{Composite}}$

Test Primes: $\cancel{2}, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{11}, \cancel{13}, \cancel{17}, \cancel{19}, 23$

$667 \div 23 = 29$

d) 211 $\sqrt{211} \xrightarrow{\text{roundup}} 15$

$\boxed{\text{Prime}}$

Test Primes: $\cancel{2}, \cancel{3}, \cancel{5}, \cancel{7}, \cancel{11}, \cancel{13}$

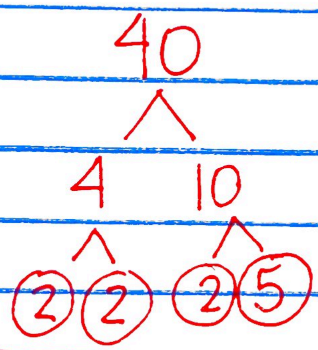
e) 8,245 $\sqrt{8245} \xrightarrow{\text{roundup}} 91$ Yikes!!

divisible by 5 $\boxed{\text{Composite}}$

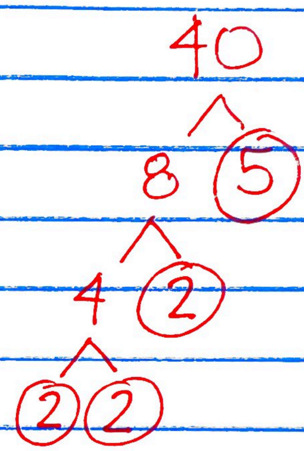
Every natural number greater than 1 can be written uniquely as a product of prime numbers. This product is called the prime factorization.

ex 3 Find the prime factorization.

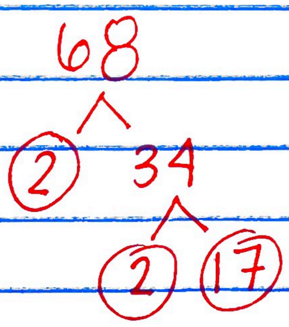
a) 40



$\boxed{2 \cdot 2 \cdot 2 \cdot 5}$



b) 68



$\boxed{2 \cdot 2 \cdot 17}$

*To check: are all #s prime?
do they multiply to be the original #?

We can use the prime factorization to generate all of the factors of a number.

For ex: $40 = 2 \cdot 2 \cdot 2 \cdot 5$

Factors of 40:

40 has 8 factors

1
$2 = 2$
$4 = 2 \cdot 2$
$5 = 5$
$8 = 2 \cdot 2 \cdot 2$
$10 = 2 \cdot 5$
$20 = 2 \cdot 2 \cdot 5$
$40 = 2 \cdot 2 \cdot 2 \cdot 5$

These are all the combinations of 2, 2, 2 and 5.

If the prime factorization of x is $p^a \cdot p^b \cdot p^c \cdots p^z$, the number of factors of x will be $(a+1)(b+1)(c+1)\cdots(z+1)$.
[Note: p must be prime.]

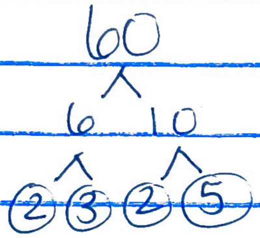
So, going back to $40 = 2^{3} \cdot 5^{1}$

the # of factors would be $(3+1)(1+1)$
$= 4 \cdot 2$
$= 8$

end 4-2

ex 4 How many factors do the following numbers have? Do not list them out.

a) 60



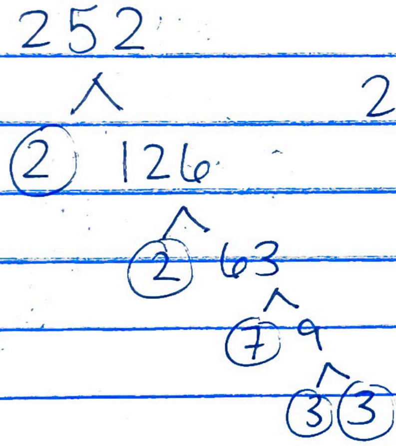
$60 = 2^{2}\cdot 3^{1}\cdot 5^{1}$

$(2+1)(1+1)\cdot(1+1)$

$= 3\cdot 2\cdot 2$

$= \boxed{12}$

b) 252



$252 = 2^{2}\cdot 3^{2}\cdot 7^{1}$

$(2+1)(2+1)\cdot(1+1)$

$= 3\cdot 3\cdot 2$

$= \boxed{18}$

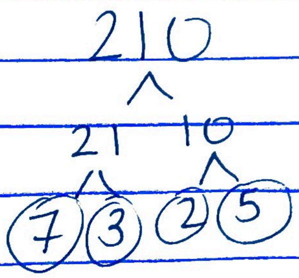
c) 1,000,000

$1{,}000{,}000 = 10^{6} = (2\cdot 5)^{6} = 2^{6}\cdot 5^{6}$

$(6+1)\cdot(6+1)$

$= \boxed{49}$

d) 210^{10}



$210 = (2\cdot 3\,5\,7)^{10}$

$= 2^{10}\cdot 3^{10}\cdot 5^{10}\cdot 7^{10}$

$= (10+1)\cdot(10+1)(10+1)(10+1)$

$= \boxed{14{,}641}$