

Math 200 Section 1-1 Problem Solving

Common Core summary

George Polya's 4 Steps for Problem Solving

What is considered a problem varies from person to person. Formally, a problem is a situation in which a solution is not apparent.

ex 1 Gerald, Trey, and Darnell ordered a large pizza. Gerald will eat three times as much pizza as Trey. Trey will eat twice as much pizza as Darnell. If the pizza is cut into 36 equal slices, how many slices will each person get?

Make a Table / Guess + Check

Gerald $\xleftarrow{\times 3}$	Trey $\xleftarrow{\times 2}$	Darnell	Does the Total = 36?
6	2	1	too small NO
18	6	3	too small NO
30	10	5	too big NO
24	8	4	yes

George Polya's Four Steps to Problem Solving

1. Understand the Problem

- a. Can you state the problem in your own words?
- b. What are you trying to find or do?
- c. What are the unknowns?
- d. What information do you obtain from the problem?
- e. What information, if any, is missing or not needed?

2. Devise a Plan

The following list of strategies, although not exhaustive, is very useful:

- a. Look for a pattern.
- b. Examine related problems and determine if the same technique applied to them can be applied here.
- c. Examine a simpler or special case of the problem to gain insight into the solution of the original problem.
- d. Make a table.
- e. Make a diagram.
- f. Write an equation.
- g. Use guess and check.
- h. Work backward.
- i. Identify a subgoal.
- j. Use indirect reasoning.
- k. Use direct reasoning.

3. Carry out the Plan

- a. Implement the strategy or strategies in step 2 and perform any necessary actions or computations.
- b. Check each step of the plan as you proceed. This may be intuitive checking or a formal proof of each step.
- c. Keep an accurate record of your work.

4. Look Back

- a. Check the results in the original problem. (In some cases, this will require proof.)
- b. Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable? Does it answer the question that was asked?
- c. Determine whether there is another method of finding the solution.
- d. If possible, determine other related or more general problems for which the techniques will work.

Common Core Mathematics

Research studies of mathematical education in high-performing countries have concluded that mathematics education in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country.

The Common Core concentrates on a clear set of math skills and concepts. These skills are designed to address the problem of a curriculum that is "a mile wide and an inch deep." Students will learn concepts in a more organized way both during the school year and across grades. The standards encourage students to solve real-world problems. Mathematical understanding and procedural skill are equally important.

ex 2 Rework ex 1 and use an equation

Let $x = \#$ pieces Darnell eats

$$\begin{array}{ccc} \swarrow D & \swarrow T & \swarrow G \\ x + 2x + 3(2x) = 36 \end{array}$$

$$x + 2x + 6x = 36$$

$$9x = 36$$

$$x = 4$$

So, Darnell eats 4 pieces
Trey eats $2(4) = 8$ pieces
and Gerald eats $3(2(4)) = 24$ pieces

ex 3 Suppose the following equation appears on the Basic Skills Test for you to solve: $x + 2x + 3(2x) = 36$.
The multiple choice answers are:

- a) $x = 1$
- b) $x = 8$
- c) $x = 2$
- d) $x = 4$

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How can you work backwards to solve this problem?

You can plug each answer in until you find the one that works!

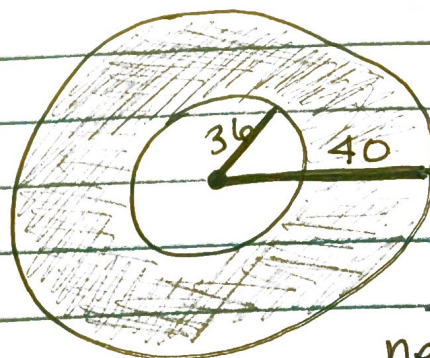
$$x + 2x + 3(2x) = 36$$

- a) $x = 1$ Is $1 + 2(1) + 3(2(1)) = 36$? No
b) $x = 8$ Is $8 + 2(8) + 3(2(8)) = 36$? No
c) $x = 2$ Is $2 + 2(2) + 3(2(2)) = 36$? No
d) $x = 4$ Is $4 + 2(4) + 3(2(4)) = 36$? Yes !!

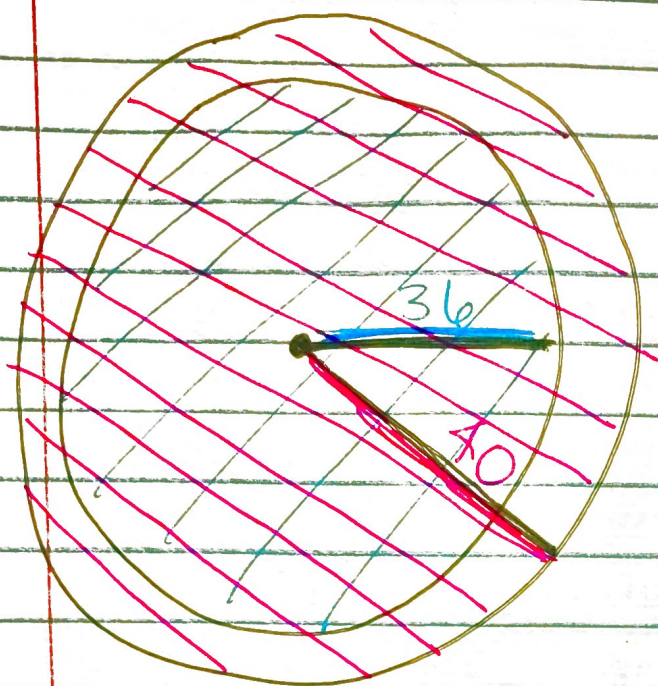
ex 4 How much area is available for a circular walk if the outer part of the walk is determined by a circle of radius 40 ft and the inner part of the walk is determined by a circle of radius 36 ft? (The circles are concentric.)

Draw a picture

not drawn
to scale ☺



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I tried to draw it more to scale here.

Pink = 40ft radius
Area = $\pi(40)^2$

Blue = 36ft radius
Area = $\pi(36)^2$

The circular walkway is formed by taking the pink area and removing the blue area.

So the circular walkway's area is:

$$\pi(40)^2 - \pi(36)^2 \approx \boxed{955 \text{ ft}^2}$$

ex 5 Find the sum.

$$1 + 2 + 3 + 4 + \dots + 97 + 98 + 99 + 100$$

Look for a Pattern

$$1 + 2 + 3 + 4 + \dots + 97 + 98 + 99 + 100$$

101

101

101

101

$$\underbrace{|0| + |0| + |0| + \dots + |0|}$$

50 times (bc 50 pairs)

So, $101(50) = 5050$

* This technique is credited to the famous mathematicians Carl Gauss. Rumor has it that this problem was given to him in elementary school in order to keep him busy. He figured it out almost immediately.

ex 6 Find the sum.

$$2 + 4 + 6 + 8 + \dots + 96 + 98 + 100$$

Examine a related problem

Using the Gaussian method from the previous example:

$$2 + 4 + 6 + 8 + \dots + 96 + 98 + 100$$


Diagram illustrating the pairing of terms in the arithmetic series:

- Innermost pair: $6 + 98 = 102$
- Middle pair: $4 + 100 = 102$
- Outermost pair: $2 + 100 = 102$

$$102 + 102 + 102 + \dots + 102$$

How many pairs?

Well, there are 50 #s in the sum, so there are 25 pairs.

So, $(102)(25) = 2550$

end of 1-1