Section 5-2 Mult + Division of Integers

Properties of Integer Multiplication

1. Closure property
The product of any 2 integers is always an integer.

* Note: The integers are not closed for division since not every quotient of 2 integers is an integer. ex: 3/4

2. Commutative property
$-3 \cdot 4 = 4 \cdot -3$

3. Associative property
$-4 \cdot (2 \cdot 5) = (-4 \cdot 2) \cdot 5$

4. Multiplicative Identity property
$-7 \cdot 1 = -7$ and $1 \cdot -7 = -7$
* Note: 1 is called the multiplicative identity element. [Identity = identical = same]

5. Zero multiplication property
$-3 \cdot 0 = 0$ and $0 \cdot -3 = 0$

6. Distributive Property of Mult over Add/Su

$5(-2+7) = 5(-2) + 5(7)$

Models for Integer Multiplication

1. Patterns

$5 \cdot 2 = 10$		$-4 \cdot 2 = -8$	
$5 \cdot 1 = 5$	-5	$-4 \cdot 1 = -4$	$+4$
$5 \cdot 0 = 0$	-5	$-4 \cdot 0 = 0$	$+4$
$5 \cdot -1 = -5$	-5	$-4 \cdot -1 = 4$	$+4$
$5 \cdot -2 = -10$	-5	$-4 \cdot -2 = 8$	$+4$
$5 \cdot -3 = -15$	-5	$-4 \cdot -3 = 12$	$+4$

2. Red + Black Chips / Charged Field

* we can use repeated addition to model pos · pos, pos · neg, and neg · pos

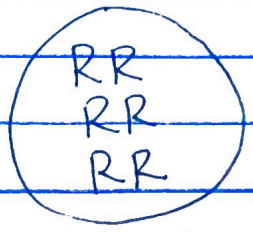
ex! Model

a) $-2 \cdot 3$

3 grps of 2 rec

commutative prop: $3 \cdot -2$

Repeated addition: $-2 + -2 + -2$



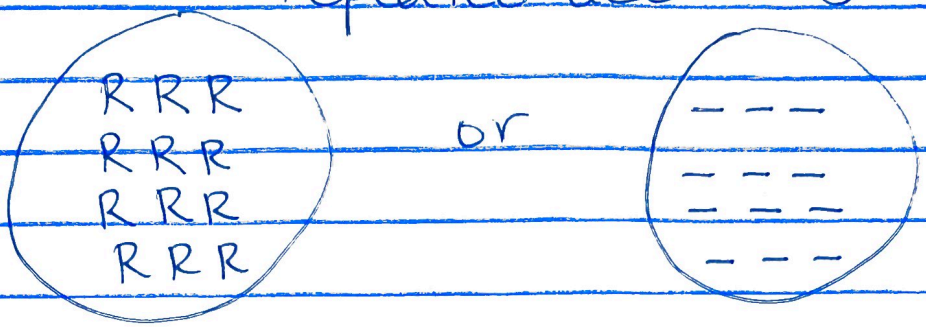
-6

red

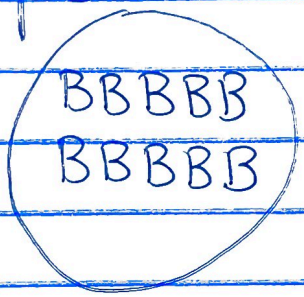
6 chips

b) $4 \cdot -3 = -12$ 4 grps of 3 red chips

repeated add: $-3 + -3 + -3 + -3$



c) $5 \cdot 2$ repeated add: $2+2+2+2+2$



* Using Black + Red chips to model negative · negative is trickier.

Look back at ex 1

a) $3 \cdot -2$ start with empty circle (zero) repeatedly add 3 groups of 2 red

↑ bc 3 is positive

↑ bc 2 is negative

b) $+4 \cdot -3$ start with empty circle (zero) repeatedly add 4 groups of 3 red

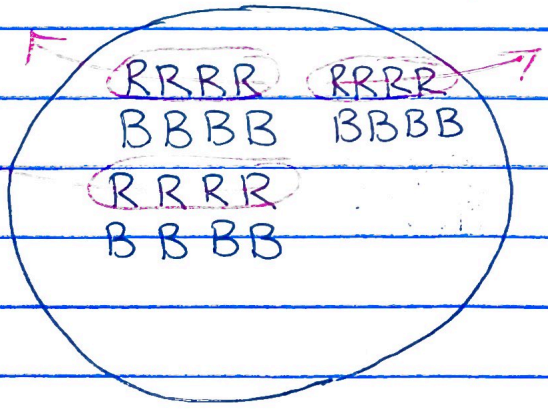
c) $+5 \cdot +2$ start with empty circle (zero) repeatedly add 5 groups of 2 blac

bc 5 is positive

bc 2 is positive

ex 2 Model $-3 \cdot -4$ using B + R chips

start with "zero", repeatedly subtract 3 grps of 4 re



left with 12 B which is +12.

So, $-3 \cdot -4 = 12$

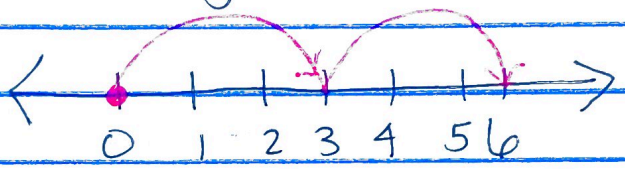
* This is weird bc the zero you start with isn't an empty circle. You have to put stuff in the circle in order to have it to remove.

3. Number Line

You can easily use a number line with pos·pos, neg·pos, and pos·neg by rewriting as repeated addition.

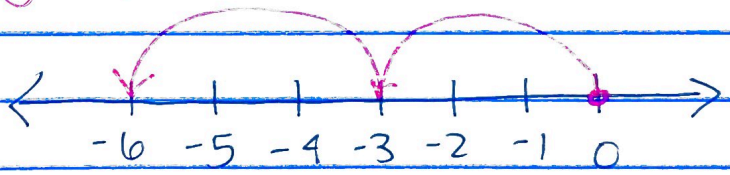
ex 3 Model using a number line.

a) $2 \cdot 3$



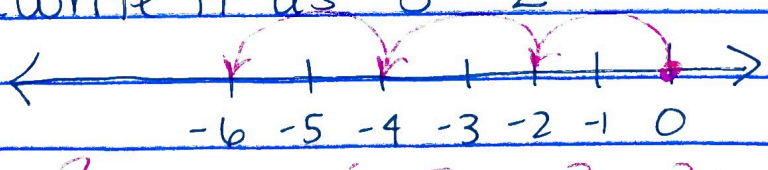
2 groups of 3 = 3 + 3

b) $2 \cdot -3$



2 groups of -3 = -3 + -3

c) $-2 \cdot 3$ use the commutative property to rewrite it as $3 \cdot -2$



3 groups of -2 = -2 + -2 + -2

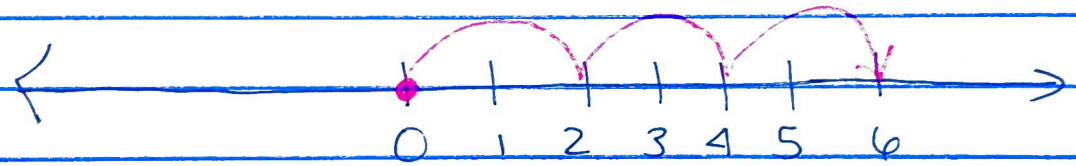
* notice how we start at zero each time.
* we can't use this method for neg·neg bc you can't have a negative # of groups.

Let's consider a word problem to examine the neg · neg case.

The temperature is now 0°. What was the temperature 3 hours ago if it has been dropping 2 degrees each hour?

This is $-3 \cdot -2 = 6$

ago dropping



Sign Rules for Integer Multiplication

positive · positive = positive
pos · neg = neg
neg · pos = neg
neg · neg = pos

Integer Division Rules

pos ÷ pos = pos
pos ÷ neg = neg
neg ÷ pos = neg
neg ÷ neg = pos

related mult problem

Recall: $10 \div 5 = 2 \Rightarrow 2 \cdot 5 = 10$

pos ÷ pos = ? ⇒ ? · pos = pos
pos ÷ neg = ? ⇒ ? · neg = pos
neg ÷ pos = ? ⇒ ? · pos = neg
neg ÷ neg = ? ⇒ ? · neg = neg

The division rules are just the mult rules rewritten in division form.

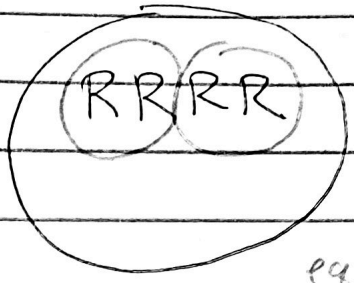
We can model integer division by using the missing factor or Black + Red chips.

* Note: The signs must be the same to use repeated subtraction. The divisor must be positive to use partitive.

ex 4 Model using Black & Red Chips

a) $-4 \div 2$ partitive

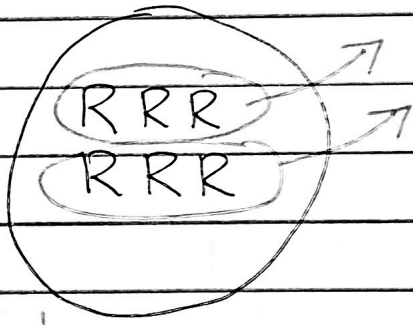
divisor is pos.



4 red split into 2 equal groups
How many are in each grp?
2 red = -2

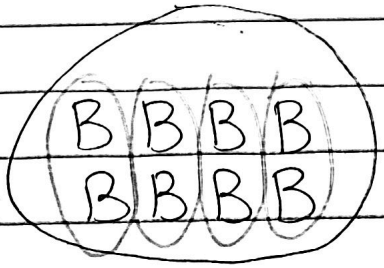
b) $-6 \div -3$ repeated subtraction

same sign

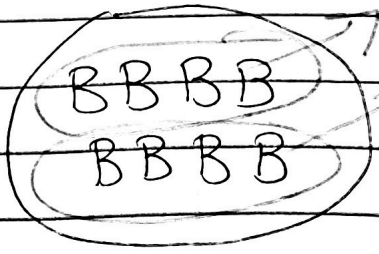


How many times can you repeatedly subtract 3R from 6R to get 0?
2 times

c) $8 \div 4$ partitive or rep sub



8 B split into 4 equal groups.
How many in each group?



How many times can you rep sub 4B from 8B?

d) $8 \div -2$ You can't use rep sub or partitive on this one.

So, we can use the missing factor model.

$8 \div -2 = \square$
is the same as

$\square \cdot -2 = 8$

Order of Operations

P	Parentheses, brackets, braces, ab. value bars	
E	exponents	
M	multiplication	if both are present, you work left to right
D	division	
A	addition	if both are present, you work left to right
S	subtraction	

$30 \div 5 \cdot 7$
$6 \cdot 7 = 42$

end 5-2