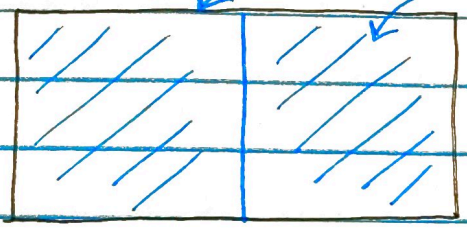
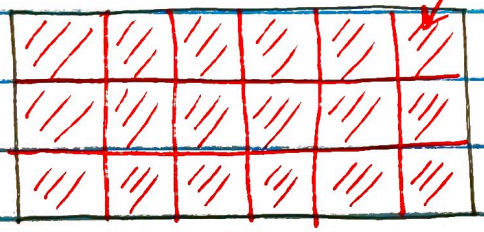
Section 13-2 Area of Polygons + Circles

We measure area in square units since the area represents a 2 dimensional shape that is being covered by non-overlapping squares that are all the same size.



each of these squares are 1 yd by 1 yd or $1\ yd^2$

Total area = $2\ yd^2$



each of these squares are 1 ft by 1 ft or $1\ ft^2$

Total area = $18\ ft^2$

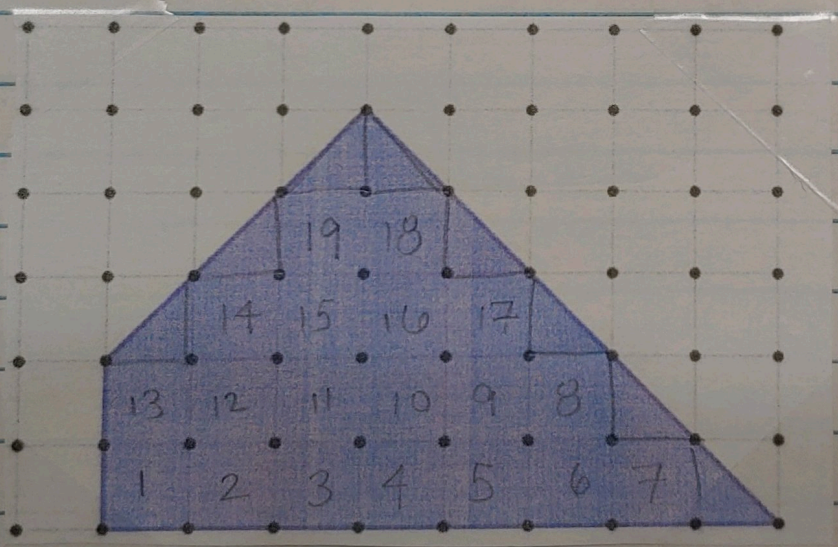
The area of both rectangles is the same; the difference is in the size of the squares (units) with which we cover the rectangle.

People mistakenly think that an area of $18\ ft^2$ is an 18 ft × 18 ft rectangle. Notice that only the unit is squared.

So, 18 ft^2 means it takes 18 non-overlapping squares that are 1 ft × 1 ft in size to cover the shape.

Let's take a small detour and talk about more hands-on activities to introduce area before we get into all of the formulas.

ex 1: Find the area of the shaded figure.



← Geoboard

Area = 19 full squares + 8 half squares

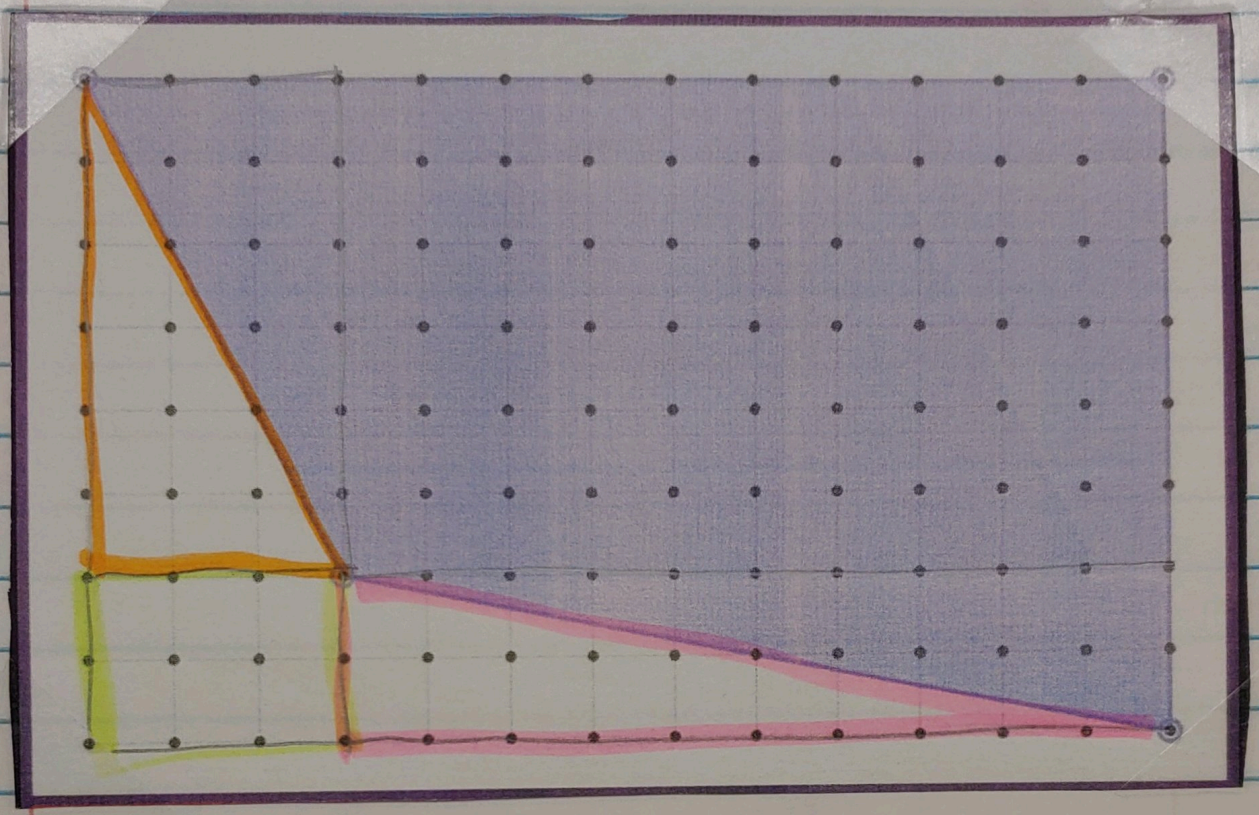
= 19 + 8 (1/2)

= 19 + 4

= **23 square units**

* Mathplayground.com has interactive geoboards.

ex 2 Find the area of the shaded figure.



In this case, it's probably easier to use subtraction.

$$A = \underbrace{(13 \cdot 8)}_{\text{Area board}} - \underbrace{\left(\tfrac{1}{2}(3 \cdot 6) + (3 \cdot 2) + \tfrac{1}{2}(2 \cdot 10)\right)}_{\text{Area of white region}}$$

$$A = 104 - (9 + 6 + 10)$$

$$A = 104 - 25$$

$$\boxed{A = 79 \text{ square units}}$$

English Units of Area

in^2 square inches
ft^2 square feet
yd^2 square yards
mi^2 square miles
acre (This unit is only for area measure, so it does not have a square on it.)

* 1 acre = $43,560\ ft^2$ (1½ football fields)

Metric Units of Area

mm^2
cm^2
dm^2
m^2
dam^2
hm^2
km^2
ha hectare (pronounced "hect-air")
a are (pronounced "air")

* $1\ a = 100\ m^2$ (a typical classroom size)
* $1\ ha = 100\ a = 10,000\ m^2$ (a little more than 2 acres or 3 football fields)

* Acre, Are, and Hectare are units of land measure.

ex 3 Convert

a) $2\,yd^2 = \_\_\_\_\_\ ft^2$

$$2\,yd^2\left(\frac{3\,ft}{1\,yd}\right)^2 = 2\,yd^2\left(\frac{9\,ft^2}{1\,yd^2}\right) = \boxed{18\,ft^2}$$

* Look back at page 1!

b) $1\,ft^2 = \_\_\_\_\_\ in^2$

$$1\,ft^2\left(\frac{12\,in}{1\,ft}\right)^2 = 1\,ft^2\left(\frac{144\,in^2}{1\,ft^2}\right) = \boxed{144\,in^2}$$

c) $870\,ft^2 = \_\_\_\_\_\ mi^2$

$$870\,ft^2\left(\frac{1\,mi}{5280\,ft}\right)^2$$

$$= 870\,ft^2\left(\frac{1\,mi^2}{5280^2\,ft^2}\right) = \boxed{0.0000312\,mi^2}$$

d) $1\ mi^2 =$ _____ acres

$$1\ \cancel{mi^2}\left(\frac{5280^2\ \cancel{ft^2}}{1^2\ \cancel{mi^2}}\right)\left(\frac{1\ acre}{43{,}560\ \cancel{ft^2}}\right) = \boxed{640\ acres}$$

e) $1/4$ acre = _____ ft^2

$$\frac{1}{4}\ \cancel{acre}\left(\frac{43{,}560\ ft^2}{1\ \cancel{acre}}\right) = \boxed{10{,}890\ ft^2}$$

ex 4 Convert

a) $5\ cm^2 =$ _____ mm^2

km	hm	dam	m	dm	cm	mm

1 place to the right
DOUBLE THAT

5.00

$\boxed{500\ mm^2}$

* To convert metric units of area, we double the number of place values moved on the stairsteps.

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b) $846\ m^2 =$ _____ km^2

m → km 3 places to the left - Double that

0.000846.

$0.000846\ km^2$

c) $72\ m^2 =$ _____ a $1a = 100m^2$

$72\ m^2 \left(\frac{1\ a}{100\ m^2}\right) = 0.72\ a$

d) $4\ ha =$ _____ km^2 $1ha = 100a$, $1ha = 10{,}000\ m^2$

$4\ ha \left(\frac{10{,}000\ m^2}{1\ ha}\right) = 40{,}000\ m^2$

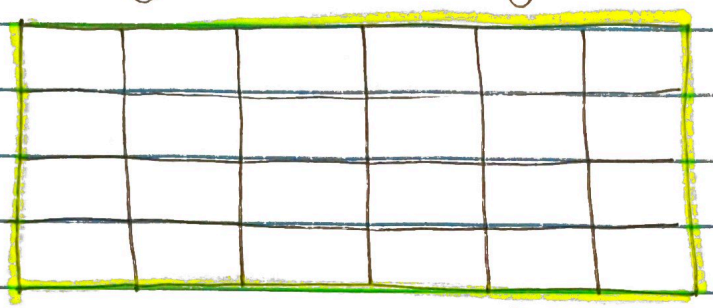
Use stairsteps m to km 3 places to the left
Double this bc $m^2 \rightarrow km^2$

0.040000.

$0.04\ km^2$

Area Formulas

Rectangle: $A = \text{length} \times \text{width}$

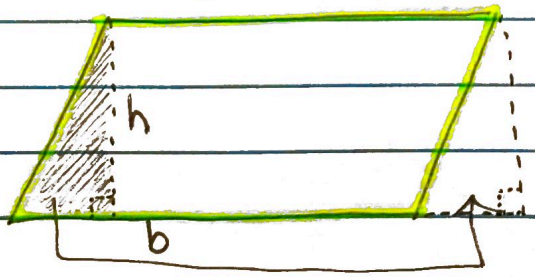


There are a total of $4 \cdot 6 = 24$ squares

Square: $A = \text{side}^2$

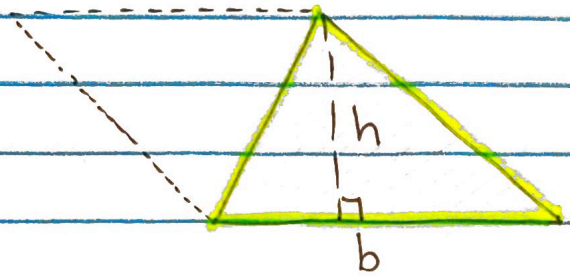
* This is a special case of a rectangle where length and width are the same.

Parallelogram: $A = \text{base} \times \text{height}$



if we cut off the shaded triangle and move it to the other end, we form a rectangle and the area of a rectangle is length × width (length = base and width = height)

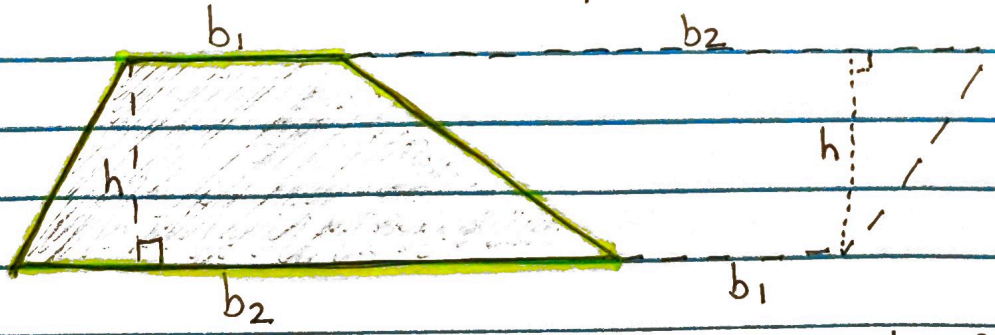
Triangle: $A = \frac{1}{2} \times \text{base} \times \text{height}$



If we copy the shaded triangle, flip it and put it next to the original triangle we form a parallelogram. The area of the parallelogram is base × height. Since the triangle is half of the parallelogram, its area is $\frac{1}{2}$(base × height).

Trapezoid: $A = \frac{1}{2} \times (\text{base } 1 + \text{base } 2) \times \text{height}$

$A = \frac{1}{2}(b_1 + b_2)h$



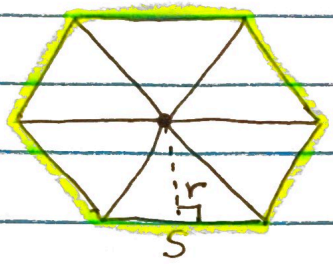
If we copy the shaded trapezoid, flip it and place it next to the original trapezoid we form a parallelogram with base length of $b_2 + b_1$ and height h. The area of this parallelogram is $(b_2 + b_1)h$ and since the

trapezoid is half of this parallelogram, its area is $\frac{1}{2}(b_2+b_1)h$.

Regular n-gon: $A = \frac{1}{2} \cdot \text{altitude} \cdot \text{perimeter}$

$A = \frac{1}{2} r P$

For ex:

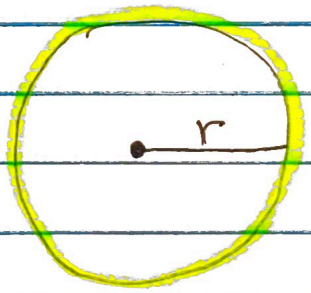


$A = \underbrace{\frac{1}{2} s r}_{\text{triangle}} \cdot \underbrace{6}_{\text{6 of them}}$

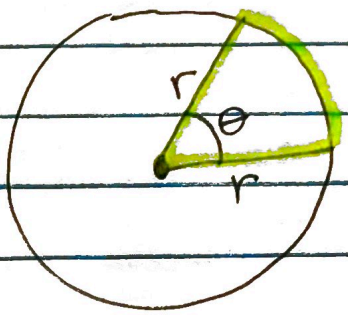
$A = \frac{1}{2} r \underbrace{(6s)}_{\text{perimeter}}$

$A = \frac{1}{2} r P$

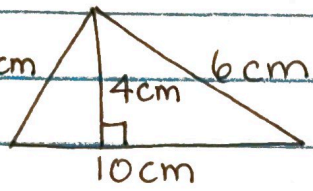
Circle: $A = \pi r^2$



Sector of a Circle: $A = \frac{\theta}{360^\circ}(\pi r^2)$

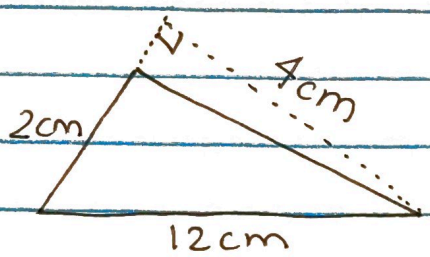


ex 5 Find the area.

a) 

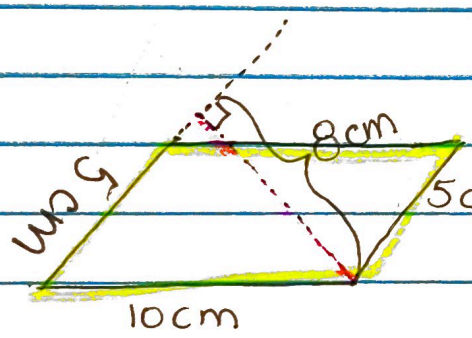
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(10)(4) = \boxed{20\ cm^2}$

b) 

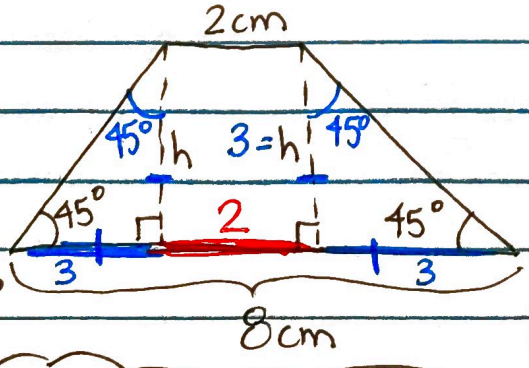
$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2)(4) = \boxed{4\ cm^2}$

c) 

$A = bh$

$A = 5(8) = \boxed{40\ cm^2}$

d) 

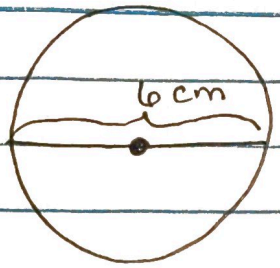
$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(2+8)(3)$

$\boxed{A = 15\ cm^2}$

$8 - 2 = 6 \div 2 = 3$

e)



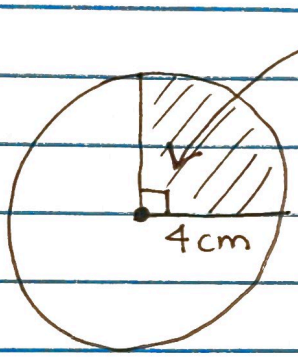
diameter = 6cm
radius = 3cm

$A = \pi r^2$

$A = \pi (3)^2$

$\boxed{A = 9\pi \text{ cm}^2}$

f)



$A = \frac{\theta}{360} \pi r^2$

$A = \frac{90}{360} \pi (4)^2$

$A = \frac{1}{4} \cdot \pi \cdot 16$

$\boxed{A = 4\pi \text{ cm}^2}$

End 13-2