

Math 206 - Test 1
Chapter 9

Name Key

Show all work to receive full credit. Partial credit may be given. Scan your work and solutions and submit them into the Test 1 D2L assignment folder on or before 11:59pm

1. (5pts) A quiz has 5 multiple choice questions. Each question has 4 answer choices, of which 1 is the correct answer and the other 3 are incorrect. Suppose that you guess on all the answers.

- (a) How many ways are there to answer the 5 questions?

$$\underbrace{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}_{(+1)} = 4^5 = \boxed{1024} (+1)$$

- (b) What is the probability of getting all 5 questions correct?

$$\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{1024}} (+1)$$

- (c) What is the probability of getting at least one question correct?

$$P(\text{at least 1 correct}) = 1 - P(\text{none correct}) = 1 - \left(\frac{3}{4}\right)^5 = \boxed{0.763} (+1)$$

2. (3pts) Your English teacher asks that you read any three of the eight books on the reading list. How many choices do you have for the set of three books you read?

$$\underbrace{8}_{(+1)} C_3 = \boxed{56} (+1)$$

Since the order does not matter, we use combinations.

3. (4pts) One week before the 2017 Major League Baseball season, the four Las Vegas favorites to win the World Series were the Chicago Cubs (23:10 odds against), Boston and Cleveland (both 6:1 odds against), and the LA Dodgers (7:1 odds against). According to the oddsmaker, what was the probability of each team winning the World Series?

Team	Odds Ag	Odds Fav	Prob Fav
Cubs	23:10	10:23	$\frac{10}{10+23} = \frac{10}{33}$ (+1)
Boston + Cleveland	6:1	1:6	$\frac{1}{1+6} = \frac{1}{7}$ (+1)
Dodgers	7:1	1:7	$\frac{1}{1+7} = \frac{1}{8}$ (+1)

4. (4pts) Box 1 contains the letters MATH. Box 2 contains the letters AND. Box 3 contains the letters HISTORY. If a box is chosen at random and then a letter is chosen a random from the box, what is the probability that the letter chosen is an A?

$$\left(\frac{1}{3}\right) \left(\frac{1}{4}\right) + \left(\frac{1}{3}\right) \left(\frac{1}{3}\right) + \frac{1}{3} \left(\frac{0}{7}\right) = \frac{7}{36} \approx 0.194$$

and
 prob of choosing box 1 (+1) prob of A from box 1
 or
 prob of choosing box 2 (+1) prob of A from box 2
 and
 prob of choosing box 3 (+1) prob of A from box 3 (+1)

5. (5pts) A box of Sweet Flakes cereal contains a Pokemon toy. There are 5 different toys to collect. Explain how you would use a random digit table or the random digit generator on your calculator to determine the number of cereal boxes that you would need to purchase in order to get all 5 toys. (You don't actually have to do the simulation, just explain how you would conduct the simulation.)

- (+1) Label the toys as 1-5. Use randInt(1,5) on calc.
- (+1) Run the simulation until you get each number 1-5.
- (+1) Count the number of trials needed to get 1-5.
- (+1) Run the simulation again (the more times the better)
- (+1) Average the number of trials per simulation.

6. (4pts) Find the number of ways to rearrange the letters of the word ANTIDIFFERENTIATION.

$$\begin{array}{ll}
 n = 19 & \# T = 3 \\
 \# A = 2 & \# F = 2 \\
 \# N = 3 & \# I = 4 \\
 & \# E = 2
 \end{array}$$

$$\frac{19!}{2! \cdot 3! \cdot 3! \cdot 2! \cdot 4! \cdot 2!} = 1.76 \times 10^{13}$$

7. (3pts) In theory, Jenny's pet frog, Phil, could hop on a keyboard and eventually type some English words. If Phil is on a keyboard with 47 keys, what is the probability that 4 random keystrokes from him will produce the word "frog"?

(+2) $\frac{1}{47} \cdot \frac{1}{47} \cdot \frac{1}{47} \cdot \frac{1}{47} = \left(\frac{1}{47}\right)^4 = \frac{1}{4,879,681} \approx 2.05 \times 10^{-7}$

prob F prob R prob O prob G

8. (8pts)

- (a) Find the probability of drawing an ace from a standard 52 card deck.

4 aces $\rightarrow \frac{4}{52} = \frac{1}{13}$ (+1)

52 total cards (+1)

- (b) Find the probability of drawing an ace from a mixture of 5 standard 52 card decks.

4 · 5 = 20 aces $\rightarrow \frac{20}{260} = \frac{1}{13}$ (+1)

52 · 5 = 260 cards

- (c) Find the probability of drawing an ace from a mixture of 1000 standard 52 card decks.

4 · 1000 = 4000 aces $\rightarrow \frac{4000}{52000} = \frac{1}{13}$ (+1)

52 · 1000 = 52000 cards

- (d) Explain any similarities or differences in your answers for parts (a), (b) and (c).

The ratio of aces to total number of cards stays the same, so the probability is the same. (+2)

9. (3pts) The probability in favor of being struck by lightning is 0.01. What are the odds in favor of being struck by lightning?

Prob in favor = 0.01 = $\frac{1}{100}$ (+1)

Odds in favor = 1 to 99 (+2)

10. (4pts) At a local racetrack, there is a wagering device called a "pick six". To win, you have to pick the winners of 6 different races. In each race there are 8 possible winners. The jackpot prize continues to grow until someone wins.

(a) Suppose the jackpot reaches \$652,195 and the tickets are \$2 each. Is it possible for you to wager in such a way as to **guarantee** winning the jackpot? Explain your answer.

There are $8^6 = 262,144$ ways for 8 horses to win 6 different races. If you buy a ticket for each of these 262,144 outcomes, you will guarantee a win.

(b) If the guarantee is profitable by how much would you profit? If it is not profitable, explain why.

Each ticket is \$2 and you must buy 262,144 tickets

$$2(262,144) = \$524,288 \quad \text{cost to buy all tickets}$$

$$\text{Win} - \text{Cost} = \text{Profit}$$

$$\$652,195 - \$524,288 = \$127,907$$

11. (3pts) Five evenly matched pigs (Apple, Boba Fett, Christmas, Dave, and Eevie) run in a race. Find the probability that Dave finishes first and Boba Fett finishes second in the race.

This problem can be done in 2 ways.

$$5P_2 = 20 \text{ outcomes for 1st + 2nd place}$$

$$\text{Prob} = \frac{1}{20}$$

$$\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}$$

$$= \frac{1}{20}$$

12. (4pts) Ming and Ji are playing a game in which they take turns rolling two 6 sided fair dice. Ming earns a point if the sum is less than 7. Ji earns a point if the sum is more than 7. They both earn a point if the sum is exactly 7. Is this game fair? Mathematically justify your yes or no answer.

sum	2	3	4	5	6	7	8	9	10	11	12
prob	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

prob of sum less than 7 is $15/36$

prob of sum greater than 7 is $15/36$

$$\text{Exp. Value Ming} : \left(\frac{15}{36}\right)(1) + \frac{6}{36}(0) + \frac{15}{36}(-1) = 0$$

The game is fair.