Section 9-3 Simulations + App in Probability

Simulations are used when the actual experiment is too complex or too expensive. A simulation is a procedure in which experiments that closely resemble the actual situation are conducted repeatedly.

ex 1: Use your calculator or a random digit table to pick 5 students out of a class of 35 to be group leaders.

math → PRB
↓
5. rand Int(

randInt (1, 35)
hit enter until you get 5 different numbers.

* Note: True randomness is hard to achieve. There are naturally occurring patterns as well as physical imperfections. A random digit table or calculator is the best way to achieve randomness.

ex 2 Explain how to use your calc or a rand number table to randomly answer a 20 question test that is:

a) All True / False questions
b) All mult choice with 4 choices and only 1 correct

a) randInt (1,2)   1 = True   2 = False
hit enter 20 times and answer based on the random # given
(repeats are okay)

b) randInt (1,4)   1=a   2=b   3=c   4=d
hit enter 20 times and answer based on the random # given
(repeats are okay)

ex 3 Suppose there are 8 colors of gumballs in a machine.

a) Use your calc or a rand digit table to find about many quarters I will need to use before getting a tasty pink gumball.

b) Use your calc or table to find the experimental probability of getting a pink gumball. (The theoretical probability is 1/8.)

a) randInt (1, 8) let's say pink = 2

1st simul : 3 tries to get a 2

2nd simul : 1 " "

3rd simul : 23 " "

4th simul : 11 " "

b) $\text{Prob} = \dfrac{\#\text{ 2s appeared}}{\text{total \# tries}} = \boxed{\dfrac{4}{38}} \approx 0.105$

experimental

$\dfrac{1}{8} \approx 0.125$

Theoretical Prob

We can express probability in terms of a ratio known as odds.

Odds are probability are often mistakenly used interchangeably. The difference between them boils down to this:

A probability compares your chance of winning (or losing) to the total number of possibilities.
The odds compare your chances of winning and losing to each other.

Suppose the odds in favor of Fat Chewy winning the Weinernationals are 1 to 40. This means out of every 41 races, Fat Chewy is expected to win 1 and lose 40.

The odds in favor are the # of favorable outcomes to the # of unfavorable outcomes.

The odds against is the reciprocal of the odds in favor.

* The odds are a ratio and can be written as:

1 to 40   1:40 $\frac{1}{40}$

* If the odds in favor are 1 to 40 then the odds against are 40 to 1.

* Odds ⟶ Probability
If the odds in favor are n to m then the probability in favor is $\frac{n}{n+m}$.

and the prob against is $\frac{m}{n+m}$.

* Probability ⟶ Odds
If $P(A)$ is the probability that A occurs and $P(\bar{A})$ is the prob that A does not occur then the:

odds in favor of A are: $\frac{P(A)}{P(\bar{A})}$ or $\frac{P(A)}{1-P(A)}$

Odds against A are: $\frac{P(\bar{A})}{P(A)}$ or $\frac{1-P(A)}{P(A)}$

ex 4 You roll a fair 6 sided die.

a) Find the prob of rolling a 4 or 5.

$\boxed{\frac{2}{6} \text{ or } \frac{1}{3}}$

b) Find the odds in favor of rolling a 4 or 5.

$\frac{P(4 \text{ or } 5)}{P(\text{not } 4 \text{ or } 5)} = \frac{1/3}{2/3} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \frac{1}{2} \text{ or } \boxed{1:2}$

c) Find the odds against rolling a 4 or 5.

recip of odds in favor $\boxed{2:1}$

ex 5 What are the odds in favor of flipping a fair coin and it landing on tails?

$P(\text{Tails}) = 1/2$

odds in favor: $\frac{P(\text{Tails})}{P(\text{Not Tails})} = \frac{1/2}{1/2} = \frac{1}{1} \quad \boxed{1:1}$

ex 6 Using a standard 52 card deck,

a) find the odds in favor of drawing a king.

$\frac{P(\text{king})}{P(\text{not king})}$ or #ways to get king : #ways not to get king

$\boxed{4 : 48}$ or $\boxed{1:12}$

b) find the odds against drawing a card that is both a 7 and a club.

$\frac{P(\text{not } 7 \text{ and club})}{P(7 \text{ and club})} = \frac{51/52}{1/52} = \frac{51}{1} \quad \boxed{51:1}$

ex 7 There is a 40% chance of snow today. What are the odds against snow today?

$P(\text{Snow}) = 40\% = 0.4$

$P(\text{Not Snow}) = 60\% = 0.6$

$$\text{odds against} = \frac{P(\text{Not Snow})}{P(\text{Snow})} = \frac{0.6}{0.4} = \boxed{3:2}$$

* Note: If the first number of the odds in favor is much larger than the second, the probability is close to 1. If the first number is much smaller than the second, the prob is close to 0.

Which weiner dog has the best chance of winning:

Pupper	odds in favor
Fat Chewy	1 : 40
Oscar	5 : 2
Flash	1 : 100
Butters	25 : 2
Skippy	1 : 1

Racetracks use odds for betting purposes. The odds that tracks give are the odds <u>against</u>.

In the last list we saw the odds <u>in favor</u> of Flash winning were 1:100. This means the odds <u>against</u> Flash winning are 100:1.

If you bet on Flash and she won, you would win \$100 for every \$1 you bet plus your bet back.

So if you bet \$20 on Flash and she won, you would win:

$$100(20) + 20 = \$2020$$

If the track gave you accurate odds, then over a long period of time, your <u>expected value</u> would be:

$$E = \underset{\substack{\uparrow \\ \text{\$100 won per \$1 bet}}}{100}\underset{\substack{\uparrow \\ P(\text{win})}}{\left(1/101\right)} + \underset{\substack{\uparrow \\ \text{\$1 bet lost}}}{(-1)}\underset{\substack{\uparrow \\ P(\text{lose})}}{\left(100/101\right)} = 0$$

If $E = 0$, it is fair.
If $E > 0$, it benefits you.
If $E < 0$, it benefits the "house."

* The expected value is the average winnings over a long period of time. (It is not useful for predicting any single outcome.)

$$E = p_1 v_1 + p_2 v_2 + p_3 v_3 + \cdots + p_n v_n$$

where the p's are probabilities and v's are their related value.

ex 8 A 5 card hand is dealt from a standard 52 card deck. If the hand contains at least one king, you win \$10, otherwise you lose \$1. Is this game fair? No bc \$2.75 > 0

$$P(\text{at least one king}) = 1 - P(\text{no king})$$
$$= 1 - \left(\frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} \cdot \frac{45}{49} \cdot \frac{44}{48}\right)$$
$$\approx 0.3412$$

$$E = (P(\text{at least 1 king}))(\text{win value}) + (P(\text{no king}))(\text{lose value})$$

$$E = (0.3412)(10) + (0.6588)(\underset{\text{lose}}{-1}) = \boxed{\$2.75}$$

ex9 A game involves you rolling a fair 6 sided die. You win the number of dollars coinciding with the number facing up.

a) Is this game fair? No, it benefits me.

$E = (1/6)(1) + (1/6)(2) + (1/6)(3) + (1/6)(4) + (1/6)(5) + (1/6)(6)$

$E = 1/6(1+2+3+4+5+6) = 21/6 = \$3.50$

b) What if you pay \$2 to play? Now is it fair?

$E = (1/6)(-1) + (1/6)(0) + (1/6)(1) + (1/6)(2) + (1/6)(3) + (1/6)(4)$

$E = 1/6(-1+0+1+2+3+4) = 9/6 = \$1.50$

Still not fair

c) How much must you pay in order for the game to be fair?

look back at part a, we need to pay \$3.50 in order for the game to be fair.

end 9-3