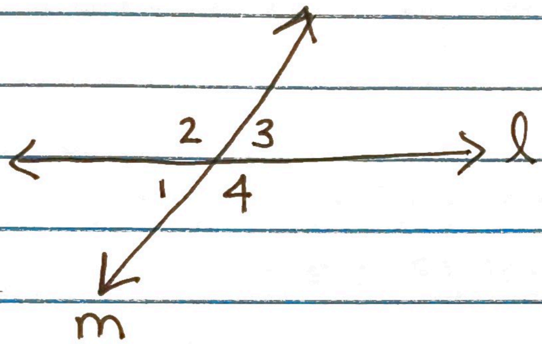
Section 11-3 More About Angles

<u>Vertical angles</u> are created by 2 intersecting lines. They are a pair of angles whose sides are 2 pairs of opposite rays.



$\angle 2$ and $\angle 4$ are vertical angles

$\angle 1$ and $\angle 3$ are vertical angles

* Vertical angles are congruent.

Proof: $\angle 1 + \angle 2 = 180^\circ$

$\angle 2 + \angle 3 = 180^\circ$

So, $\angle 1 + (180^\circ - \angle 3) = 180^\circ$

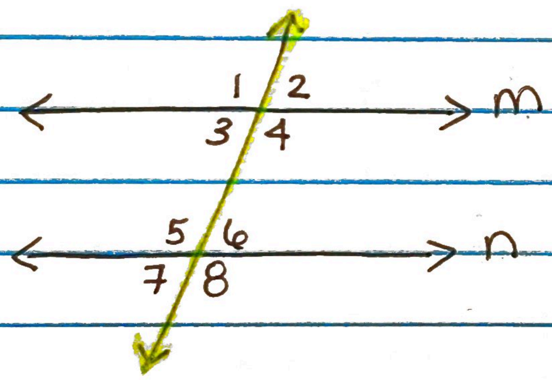
$\angle 1 - \angle 3 = 0^\circ$

$\angle 1 = \angle 3$ ∎

Two positive angles whose sum is 180° are <u>supplementary angles.</u>

Two acute angles whose sum is 90° are <u>complementary angles.</u>

Any line that intersects a pair of lines is called a <u>transversal</u> of those lines.



Interior angles: < 3, 4, 5, 6
Exterior angles: < 1, 2, 7, 8

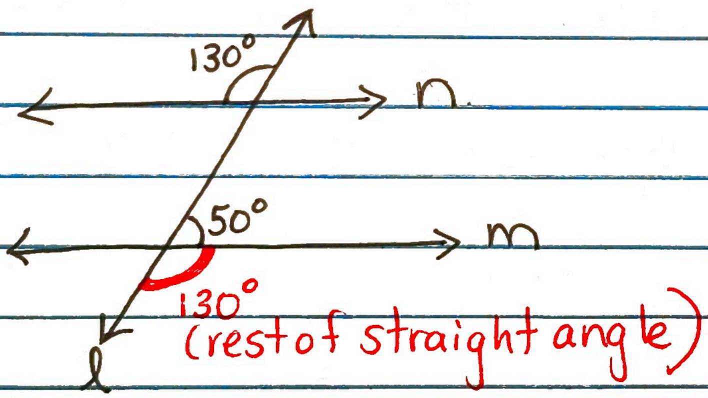
Alternate interior angles: <3 + 6, < 4 + 5
Alternate exterior angles: <2 + 7, <1 + 8

Corresponding angles: <1 + 5
<2 + 6
<3 + 7
<4 + 8

If lines m and n are parallel then corresponding angles are congruent, alt. interior angles are congruent, and alt. exterior angles are congruent.

Similarly, if all these pairs are congruent then you can deduce that m and n are parallel.

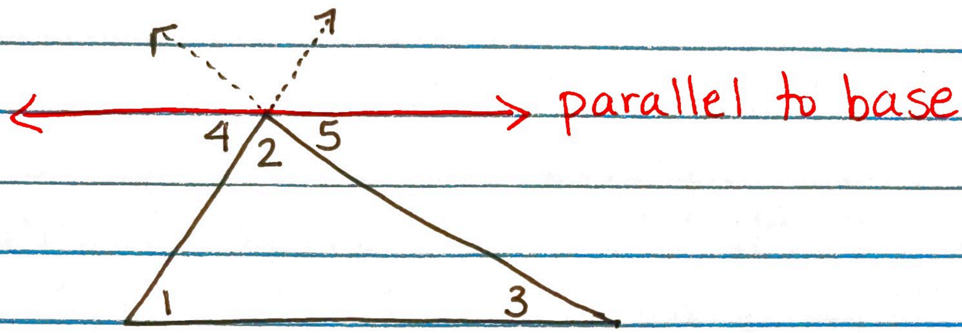
ex1 Determine if $m \parallel n$.



Since 130° = 130° (given angle)
then $m \parallel n$.

$\boxed{\text{The sum of the measures of the interior angles of a triangle is always } 180^\circ.}$

Proof:

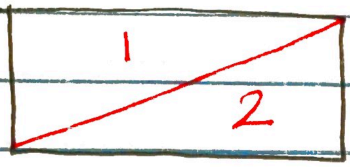


$\angle 1 = \angle 4$ alternate interior angles
$\angle 3 = \angle 5$ alternate interior angles

$\angle 4 + \angle 2 + \angle 5$ make up a straight line
So, $\angle 4 + \angle 2 + \angle 5 = 180^\circ$
↓same ↘same
So, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ ∎

We can use this fact about triangles to find the sum of the interior angles in any polygon.

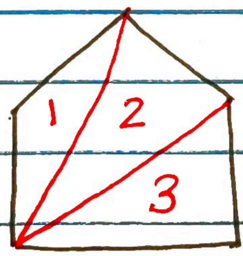
Quadrilateral



We can split any quadrilateral into 2 triangles. Each triangle has an interior angle sum of 180°, so the sum of the interior angles in any quadrilateral is:

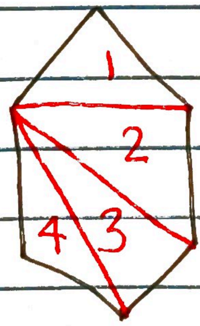
$$2 \cdot 180° = 360°$$

Pentagon



$$3 \cdot 180° = 540°$$

Hexagon



$$4 \cdot 180° = 720°$$

I form triangles by connecting a vertex with all of the non-adjacent vertices.

#sides	Polygon	(vertex) Sum of interior angles
3	Triangle	180°
4	Quadrilateral	$2 \cdot 180^\circ = 360^\circ$
5	Pentagon	$3 \cdot 180^\circ = 540^\circ$
6	Hexagon	$4 \cdot 180^\circ = 720^\circ$
7	Heptagon	$5 \cdot 180^\circ = 900^\circ$
8	Octagon	$6 \cdot 180^\circ = 1080^\circ$
9	Nonagon	$7 \cdot 180^\circ = 1260^\circ$
10	Decagon	$8 \cdot 180^\circ = 1440^\circ$
⋮	⋮	
n	n-gon	$(n-2)(180)$

ex 2 Find the measure of each vertex angle in a regular: all sides cong. + all angles cong.

a) heptagon sum $= 5 \cdot 180 = 900^\circ$

So each vertex angle measures $\frac{900^\circ}{7} = \boxed{128.6^\circ}$

b) dodecagon sum $= (12-2)(180) = 1800^\circ$

each vertex angle measures $\frac{1800^\circ}{12} = \boxed{150^\circ}$

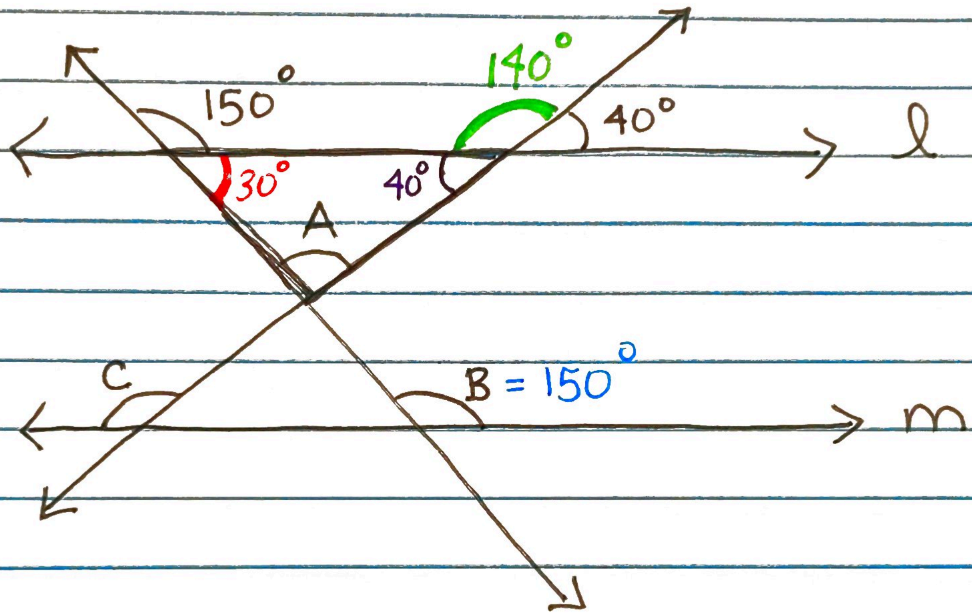
c) 52-gon sum $= (52-2)(180^\circ) = 9000^\circ$

each vertex angle measures $\frac{9000^\circ}{52} \approx \boxed{173.1^\circ}$

ex 3 Given that $l \parallel m$, find $m(\angle A)$, $m(\angle B)$, and $m(\angle C)$.

(Not drawn to scale.)

measure of angle A



Sum of angles in a triangle is 180°

$30° + 40° + \angle A = 180°$

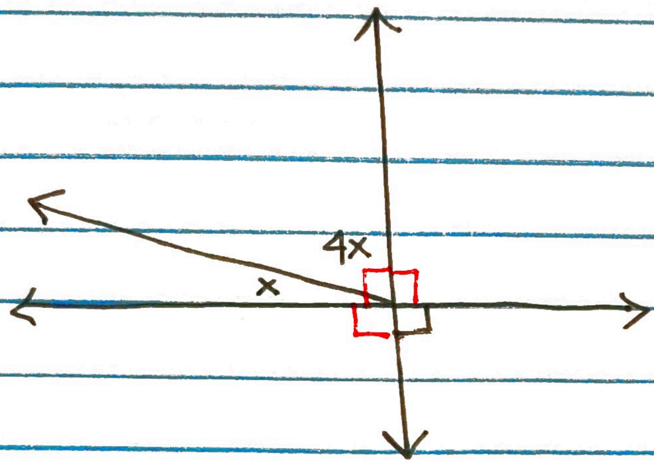
$70° + \angle A = 180°$

$\boxed{\angle A = 110°}$

$\boxed{\angle B = 150°}$ corresponding angles

$\boxed{\angle C = 140°}$ corresp angles ex 4 Solve for x.

a)

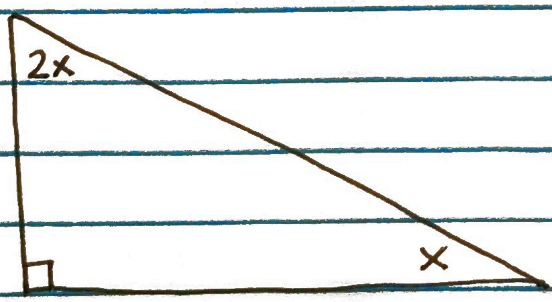


$4x + x = 90°$

$\frac{\cancel{5} \cdot x}{\cancel{5}} = \frac{90°}{5}$

$\boxed{x = 18°}$

b)



$2x + x + 90° = 180°$

$3x + \cancel{90°} = 180°$

$\quad \cancel{-90} \quad -90$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{90}{3}$

$\boxed{x = 30°}$

end 11-3