Section 6-3 Mult + Div of Rational #s

Properties of Rational Number Multiplication

1. closure
The product of any 2 rational numbers is always a rational number.

2. commutative
ex: $\frac{3}{7} \cdot \frac{1}{8} = \frac{1}{8} \cdot \frac{3}{7}$

3. Associative
ex: $\frac{1}{3} \cdot \left(\frac{1}{6} \cdot \frac{2}{9}\right) = \left(\frac{1}{3} \cdot \frac{1}{6}\right) \cdot \frac{2}{9}$

4. Multiplicative Identity
ex: $\frac{4}{7} \cdot 1 = \frac{4}{7}$ and $1 \cdot \frac{4}{7} = \frac{4}{7}$

* 1 is the multiplicative identity element

5. Zero multiplication property
ex: $\frac{5}{11} \cdot 0 = 0$

6. Multiplicative Inverse Property

ex. $\frac{2}{3} \cdot \frac{3}{2} = 1$

* the multiplicative inverse of a/b is b/a. This is also called the reciprocal.

Defn For any nonzero rational number a/b, the multiplicative inverse (or reciprocal) is the rational number b/a such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.

* 0 has no multiplicative inverse. Why?

BC there is no number that you can multiply by 0 and get 1.

7. Distributive Prop of Mult across add/sub

ex: $\frac{2}{3}\left(6 - \frac{1}{5}\right) = \frac{2}{3} \cdot 6 - \frac{2}{3} \cdot \frac{1}{5}$

Multiplication

We are used to multiplying whole numbers where the outcome is larger than either of the factors that were multiplied together. Multiplication of a whole number by a fraction between 0 and 1 produces an answer less than the whole number.

Let's look at multiplication in 2 cases:

1. whole number times rational number (that is not a whole #)
2. rational number times rational number

Case 1 Whole # times Rational

ex: $3 \cdot \frac{2}{5}$

We can use repeated addition to get:

$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{2+2+2}{5} = \frac{3 \cdot 2}{5} = \frac{6}{5}$$

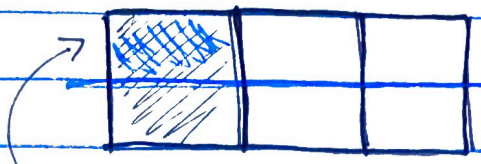
$$\frac{3}{1} \cdot \frac{2}{5}$$

where neither is a whole #

Case 2 Rational # times Rational

ex: $\frac{1}{2} \cdot \frac{1}{3}$

$\frac{1}{2}$ of $\frac{1}{3}$



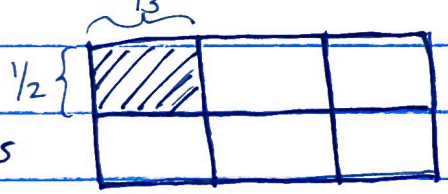
$\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$

This way isn't always nice.

ex: $\frac{1}{2} \cdot \frac{1}{3}$

rows ↑ columns ↑

Area model



* We don't need a common denominator to multiply fractions. Think about this:

$\frac{1}{2}$ of 6 feet is 3 feet

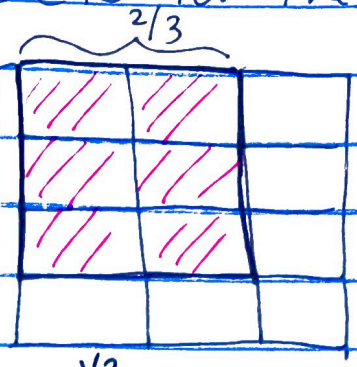
↑ we don't need the units the same like we do with addition & subtraction

ex 1 Draw area models for the following.

a) $\frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12}$

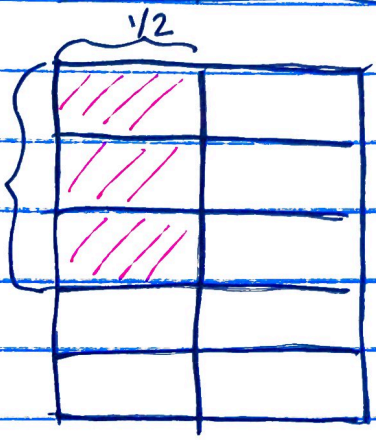
R ↑ C ↑

* denom = rows + columns



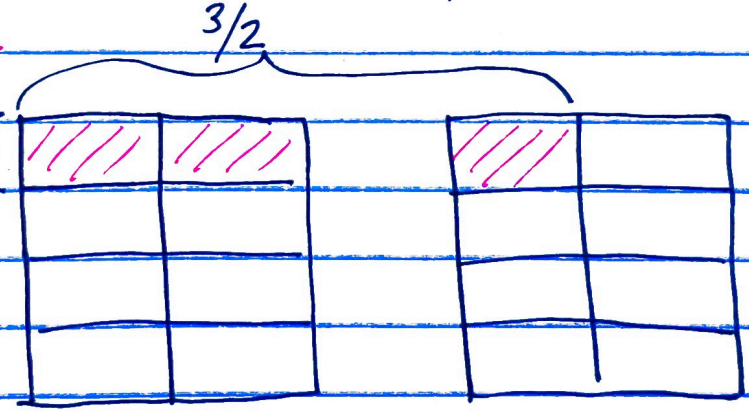
b) $\frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$

R ↑ C ↑



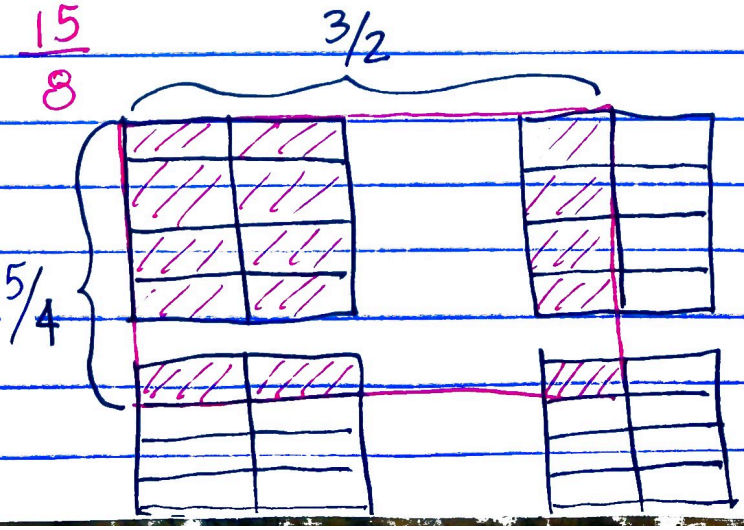
c) $\frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8}$

C ↑ R ↑



d) $\frac{5}{4} \cdot \frac{3}{2} = \frac{15}{8}$

R ↑ $\frac{1}{4}$ ↑ $\frac{1}{2}$ ↑ C



From these area models we see:

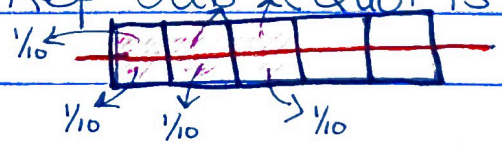
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Division

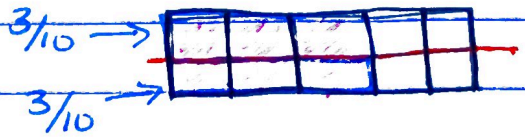
Let's try to relate our division techniques of partition, repeated subtraction, and missing factor to rational number division.

ex 2 Classify as either Partition or Rep Sub.

a) $3/5 \div 1/10 = 6$ Rep Sub (Quot is whole #)



b) $3/5 \div 2 = 3/10$ Partitive (divisor is whole #)



If neither the quotient nor the divisor is a whole number, we can use the missing factor to develop a formula.

ex: $\frac{2}{3} \div \frac{4}{5} = ? \Rightarrow ? \cdot \frac{4}{5} = \frac{2}{3}$

If you think of this like an equation:

$\frac{4}{5} x = \frac{2}{3}$

You can solve by multiplying both sides by the reciprocal of 4/5:

$\frac{5}{4} \cdot \frac{4}{5} x = \frac{2}{3} \cdot \frac{5}{4}$

$x = \frac{2 \cdot 5}{3 \ 4}$

So, $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4}$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b}(bd) \div \frac{c}{d}(bd) = ad \div cb = \frac{a \cdot d}{c \cdot b}$$

In general, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, $b, c, d \neq 0$

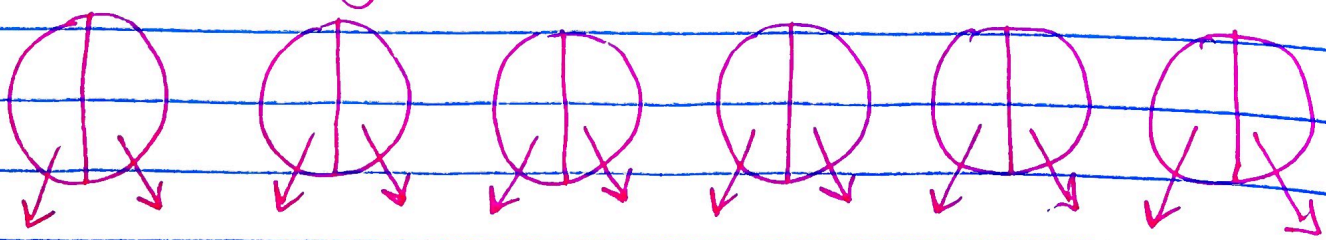
ex 3 Evaluate $1\frac{3}{4} \div \frac{1}{2}$

$$\frac{7}{4} \div \frac{1}{2} = \frac{7}{4} \cdot \frac{2}{1} = \frac{14 \div 2}{4 \div 2} = \boxed{\frac{7}{2}}$$

$1\frac{3}{4} = \frac{4 \cdot 1 + 3}{4}$

ex 4 Write an application (word) problem that models $6 \div \frac{1}{2}$.

You have 6 cookies and want to give each child ½ cookie. How many kids can get ½ of a cookie?



* Look at data of Chinese teachers versus American teachers.

end 6-3

Division by Fractions

The following is an excerpt from Liping Ma's book *Knowing and Teaching Elementary Mathematics*:

"Imagine that you are teaching division with fractions. To make this meaningful for kids, something that many teachers try to do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story-problems to show that application of some particular piece of content. What would you say would would be a good story or model for $1\frac{3}{4} \div \frac{1}{2}$?" (Ma, p.55)

The following table shows the vast difference in teacher's knowledge of division by fractions.

Teachers	Total	Correct Computation	Correct Story Problem	Multiple Story Problems
American	23	10	1	0
Chinese	72	72	65	12

Based on the above table, one might find it hard to believe that American teachers are far more educated than Chinese teachers. In fact, most Chinese teachers only complete the ninth grade before continuing on to normal school for 2 or 3 additional years of study. Liping Ma's explanation for this discrepancy is that Chinese teachers have a greater understanding of fundamental mathematics. "Indeed, about 10% of those Chinese teachers, despite their lack of fomal education, display a depth of understanding which is extraordinarily rare in the United States." (Ma,p.xvii)

The following are a few teacher responses from Liping Ma's book that show the lack of understanding of most American teachers. If the pseudonym begins with Ms. or Mr. this represents a beginning teacher while a pseudonym that begins with Tr. represents an experienced teacher. When presented with the question of calculating and coming up with a story problem for $1\frac{3}{4} \div \frac{1}{2}$ the following responses were given:

American Teachers

"The first thing you'd *have to do* is change them into sync. Well, you're supposed to multiply that and add that. So, that's 4 plus it's $\frac{7}{4}$, and then you *have to make it the same*. Divided by $\frac{2}{4}$. Right? And then you just cross multiply like that. You get $\frac{28}{8}$? (Ms. Felice, italics added)"

"It makes, it [the calculation that she did] was wrong. Because you have a half of a one would be one half, and a half of three fourths would be [lenghty pause] if you estimated it be a fourth and then a little bit more. Let's see, that the answer is a little over three fourths...When I did it in a real-world thing, I would realize that I had done it wrong, and then I'd just go over it again. When you do that without a real-world thing you might be doing them real wrong, and you might do the problem wrong that way. (Ms. Felice)"

"I would try to find, oh goodness, the lowest common denominator. I think I would change them both. Lowest common denominator, I think that is what it is called. I do not know how I am going to get the answer. Whoops. Sorry. (Tr. Bernadette)"

"You could be using pie, a whole pie, one, and then you have three fourths of another pie and *you have two people*, how will you make sure that this gets *divided evenly*, so that each person gets an equal share. (Ms. Fiona)"

Chinese Teachers

"Dividing a number is equivalent to multiplying by its reciprocal. So, to divide $1\frac{3}{4}$ by $\frac{1}{2}$ we multiply $1\frac{3}{4}$ by the reciprocal of $\frac{1}{2}$, $\frac{2}{1}$, and we get $3\frac{1}{2}$. (Ms. M)"

"Cut an apple into four pieces evenly. Get three pieces and put them together with a whole apple. Given that $\frac{1}{2}$ apple will be a serving, how many servings can we get from the $1\frac{3}{4}$ apples? (Ms. I)"

"The mom bought a box of candy. She gave $\frac{1}{2}$ of it which weighed $1\frac{3}{4}$ kg to the grandma. How much did the box of the candy originally weigh? (Ms. M)"