Section 1-2 Explorations With Patterns

Patterns are everywhere: weather, nature, traffic lights, our daily routine, ...

In this section we look at some specific types of patterns.

There are 2 types of reasoning:

1. <u>Inductive Reasoning</u> is drawing conclusions based on collected data and observed patterns (may not always be true)

2. <u>Deductive Reasoning</u> is using proven facts and logic to validate an argument.

Example of inductive reasoning:

$0^2 = 0$

$1^2 = 1$

Therefore any number squared is itself.

This conclusion is called a conjecture and it is either proved by deductive reasoning or disproved by a counterexample.

Counterexample: $4^2 = 16$

* It only takes one counterexample to disprove a conjecture.

We will look at patterns inductively.

ex1 Find the next 2 terms.

a) 1, 5, 9, 13, 17, 21

+4 +4 +4 +4 +4

b) 29, 24, 19, 14, 9, 4

−5 −5 −5 −5

c) 1, 2, 4, 5, 7, 8, 10, 11, 13, 14

+1 +2 +1 +2 +1 +2 +1 +2 +1

d) 3, 9, 27, 81, 243, 729

×3 ×3 ×3 ×3 ×3

e) 1, 4, 9, 16, 25, 36, 49

+3 +5 +7 +9 +11 +13

The sequences in parts a and b above are called arithmetic sequences.

Perfect squares

$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, \ldots$

An <u>arithmetic sequence</u> is formed by adding a fixed number, called the common difference, to each term to obtain the next term.

n	a_n	formula
1^{st}	1	a_1
2^{nd}	5	$a_1 + d$
3^{rd}	9	$a_1 + 2d$
4^{th}	13	$a_1 + 3d$
5^{th}	17	$a_1 + 4d$
⋮		⋮
100^{th}		$a_1 + 99d$

* note: $d = 4$

Formula: $a_n = a_1 + (n-1)d$

a_n: n^{th} term; a_1: 1^{st} term; n: term # ($1^{st}, 2^{nd}, 3^{rd}, \ldots$); d: common difference

<u>ex2</u> Find the 100^{th} term for the arithmetic sequence 1, 5, 9, 13, 17, ...

Formula: $a_n = a_1 + (n-1)d$ 4

$a_{100} = 1 + (100-1) \cdot 4$ use calc

$\boxed{a_{100} = 397}$

ex3 How many terms are in the arithmetic sequence:

$$29, 24, 19, 14, \ldots, -401$$

$d = -5$ from ex 1 b

$$a_n = a_1 + (n-1)d$$

$$-401 = 29 + (n-1) \cdot -5$$

$$-401 = 29 + -5n + 5$$

$$-401 = 29 - 5n + 5$$

$$-401 = \cancel{34} - 5n$$
$$-34 \quad \cancel{-34}$$

$$\frac{-435}{-5} = \frac{\cancel{-5}n}{\cancel{-5}}$$

$-435 \div -5$

$$\boxed{87 = n}$$

ex 4 Find the sum $2+5+8+\cdots+296$.

We could use the Gauss method here, but it is really messy. The Gauss method works on arithmetic sequences so we can use a shortcut:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

sum of the first n terms of an arith. seq

terms

1st term

last term

Before we can use this, we need to know n (the number of terms in the sequence). We can find this using

$a_n = a_1 + (n-1)d$.

$$2 + 5 + 8 + \cdots + 296$$

a_1 +3 $d=3$ a_n

$$a_n = a_1 + (n-1)d$$

$$296 = 2 + (n-1)3$$

$$296 = 2 + (n-1)3$$

$$296 = 2 + 3n - 3$$

$$296 = 3n - \cancel{1}$$

$$+1 \qquad\qquad +\cancel{1}$$

$$\frac{297}{3} = \frac{\cancel{3}n}{\cancel{3}}$$

$$99 = n$$

Sum: $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{99} = \frac{99}{2}(2 + 296)$$

$$\boxed{S_{99} = 14,751}$$

A geometric sequence is formed by multiplying by the same nonzero number, called the common ratio.

From ex1 d $3, 9, 27, 81, \ldots$
$\times 3 \quad \times 3 \quad \times 3$

ex5 Identify the following as arithmetic, geometric, or neither.

a) $18, 30, 42, 54, \ldots$ arithmetic $d = 12$
$+12 \quad +12 \quad +12$

b) $1, 3, 6, 10, 15, \ldots$ neither
$+2 \quad +3 \quad +4 \quad +5$

c) $-3, 6, -12, 24, \ldots$ geometric $r = -2$
$\times -2 \quad \times -2 \quad \times -2$

d) $5, 5, 5, 5, \ldots$ arithmetic + geometric
$+0 \quad +0 \quad +0$
$d = 0$ $r = 1$

e) $1/2, 3/8, 9/32, 27/128, \ldots$ geometric
$r = 0.75$ or $\frac{3}{4}$

f) $3/2, 7/4, 2, 9/4, \ldots$ arithmetic $d = 0.25$
or $d = 1/4$

g) $1, 1, 2, 6, 24, 120, \ldots$ neither
$\times 1 \quad \times 2 \quad \times 3 \quad \times 4 \quad \times 5$

h) $9, 7, 5, 3, 1, \ldots$ arithmetic
$-2 \quad -2 \quad -2 \quad -2$
$d = -2$

Other Sequences

Figurate Numbers

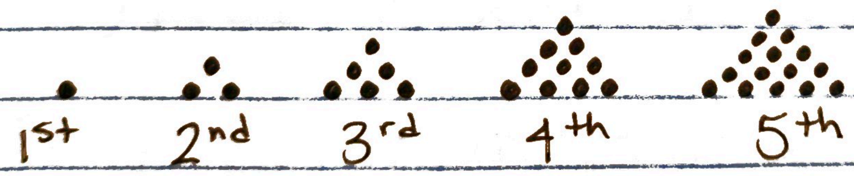
Triangular Numbers: 1, 3, 6, 10, 15, ...

Square Numbers: 1, 4, 9, 16, 25, ...

ex 6 Find the 250^{th} triangular number.

1, 3, 6, 10, 15, ... is not arithmetic so we can't use $a_n = a_1 + (n-1)d$.

Let's look for another pattern.



$1^{st} = 1$

$2^{nd} = 1 + 2$

$3^{rd} = 1 + 2 + 3$

$4^{th} = 1 + 2 + 3 + 4$

$5^{th} = 1 + 2 + 3 + 4 + 5$

So, $250^{th} = 1 + 2 + 3 + 4 + \cdots + 250$

To add this up we can use

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Since $1 + 2 + 3 + 4 + \cdots + 250$ is arithmetic

a_1 a_n

$n = 250$ since the terms increase by 1 each time

$$S_n = \frac{250}{2}(1 + 250) = \boxed{31{,}375}$$

Fibonacci Sequence

$1, 1, 2, 3, 5, 8, 13, 21, \ldots$

This is a recursive sequence that is formed by adding the 2 previous terms to get the next term.