Section 9-2 Multi-Stage Experiments Pg 1/6

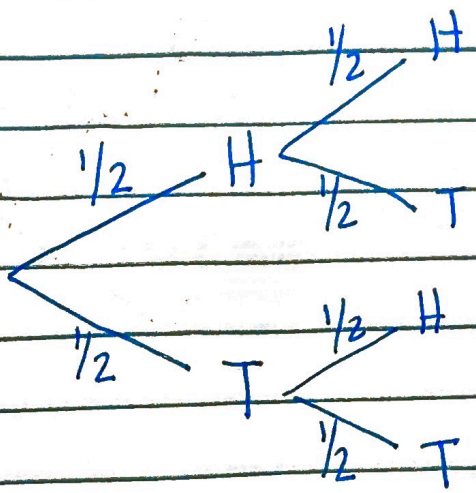
In section 9-1 we looked at simple events (also called one-stage experiments).

In this section we look at multi-stage experiments (also called compound events).

We can use a tree diagram to give us a visual of a multi-stage experiment. Trees are also helpful since there is a multiplication rule that says:

> For all multi-stage experiments, the prob of an outcome is the product of all the prob along the path to that outcome.

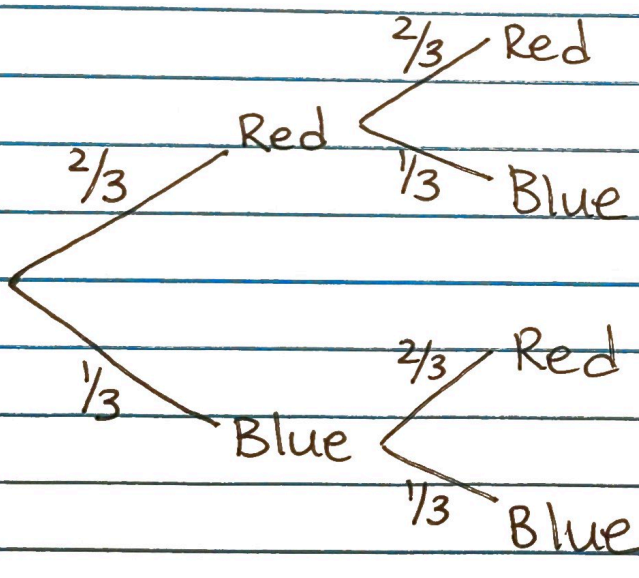
ex 1 Draw a tree diagram for flipping a fair coin twice.



Suppose you have a jar that contains 2 red marbles and 1 blue marble.

Draw a marble, record the color, replace it, then draw again.

The sample space would be: RR, RB, BR, BB



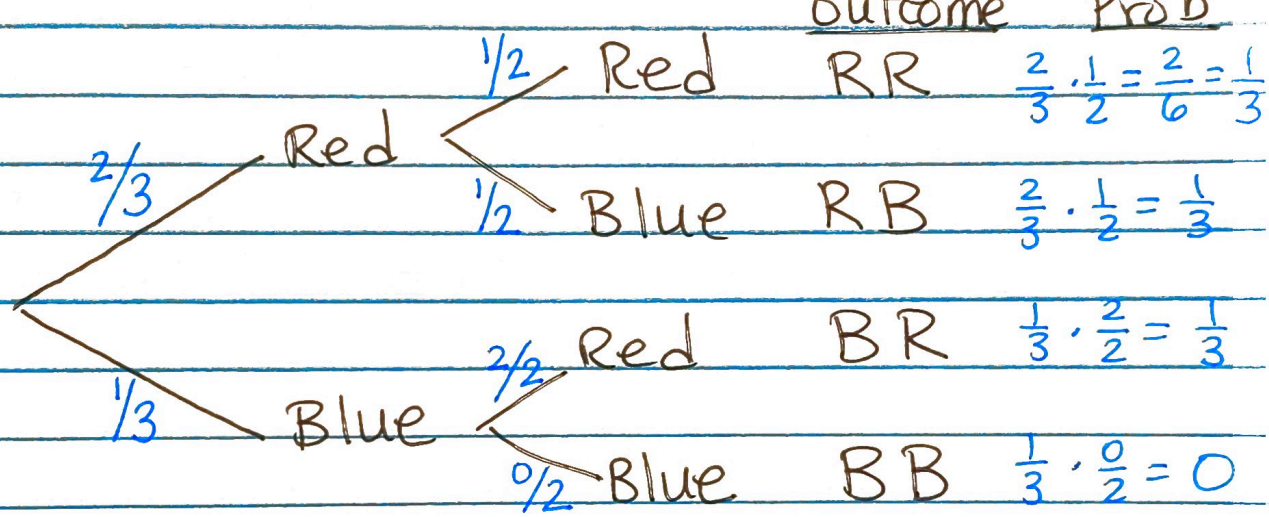
outcome	Prob
RR	$\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$
RB	$\frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$
BR	$\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$
BB	$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

* Note: $P(S) = 1$ So, $4/9 + 2/9 + 2/9 + 1/9 = 1$.

In this example, the first marble drawn is the first event and the second marble drawn is the second event.

Since the first event's outcome had no effect on the second event, we say the 2 events are independent.

ex 2 Rework the previous marble draw but NOT replace the first marble. How do the probabilities change?



outcome	Prob
RR	$\frac{2}{3}\cdot\frac{1}{2}=\frac{2}{6}=\frac{1}{3}$
RB	$\frac{2}{3}\cdot\frac{1}{2}=\frac{1}{3}$
BR	$\frac{1}{3}\cdot\frac{2}{2}=\frac{1}{3}$
BB	$\frac{1}{3}\cdot\frac{0}{2}=0$

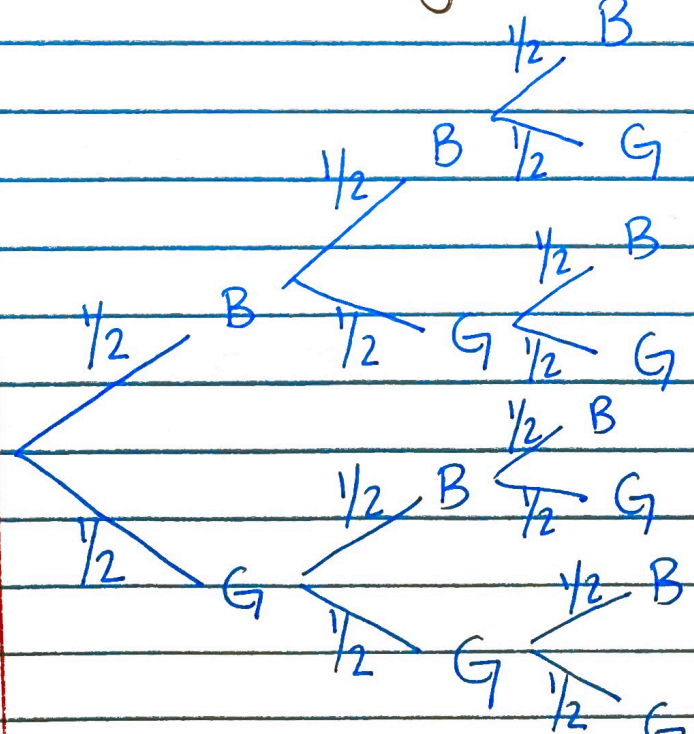
* Since we are not replacing the first marble, the first draw affects the second. So these 2 events are dependent.

Replace marble $\Rightarrow$ independent
Don't replace marble $\Rightarrow$ dependent

When working with probabilities remember:
"and" = multiply
"or" = add

ex 3 A couple plan to have 3 children. Draw a tree to show all of the possible outcomes. Then use the tree to find the probability of:

a) boy - girl - girl (in that order)
b) 2 girls and 1 boy (in any order)
c) 3 boys
d) at least 1 girl



outcomes	Prob
BBB	1/8
BBG	1/8
BGB	1/8
BGG	1/8
GBB	1/8
GBG	1/8
GGB	1/8
GGG	1/8

a) 1/8 b) 3/8 c) 1/8 d) 7/8

ex 4 You draw 2 cards from a standard 52 card deck without replacement. Find the prob that the cards are:

a) both hearts $\frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} = \boxed{\frac{1}{17}}$

b) both face cards $\frac{12}{52} \cdot \frac{11}{51} = \frac{132}{2652} = \boxed{\frac{11}{221}}$

c) both red cards $\frac{26}{52} \cdot \frac{25}{51} = \frac{650}{2652} = \boxed{\frac{25}{102}}$

d) both 3s $\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \boxed{\frac{1}{221}}$

e) an ace then a 7

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = \boxed{\frac{4}{663}}$$

ex 5 If 4 letters are randomly selected from the word ALABAMA, find the probability that the letters spell LAMB.

a) with replacing letters

L A M B

$\frac{1}{7} \cdot \frac{4}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \boxed{\frac{4}{2401}}$

b) without replacing letters

L A M B

$\frac{1}{7} \cdot \frac{4}{6} \cdot \frac{1}{5} \cdot \frac{1}{4} = \frac{4}{840} = \boxed{\frac{1}{210}}$

* mashupmath.com has a cool prob activity using candy to build an area model for outcomes then use it to find prob. (video is 7:26 long + linked on my website)

end 9-2