Section 7-3 Repeating Decimals

Recall from 7-1 that if the prime factorization of the denominator of a simplified rational number contains only 2s and/or 5s then it will be a terminating decimal.

Otherwise it will be a repeating decimal.

ex 1 Convert $3/11$ to a decimal.

$$\begin{array}{r} .2727\ldots \\ 11 \enclose{longdiv}{3.0000} \\ -22 \\ \hline 80 \\ -77 \\ \hline 30 \\ -22 \\ \hline 80 \\ -77 \\ \hline 3 \end{array}$$

So, $3/11 = 0.2727\ldots$

This can be written as $0.\overline{27}$

In the repeating decimal $0.\overline{27}$ the 27 is called the <u>repetend</u> and we say it has a period of 2.

* Note: $0.\overline{27} \neq 0.2\overline{7}$

$0.272727\ldots \neq 0.2777\ldots$

<u>ex 2</u> Convert $0.\overline{4}$ to a rational number.

Let $x = 0.4444\ldots$

So $10x = 4.444\ldots$

$$\begin{array}{rcl} 10x & = & 4.444\ldots \\ -\ 1x & = & 0.444\ldots \\ \hline 9 \cdot x & = & 4 \end{array}$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{4}{9}$$

$$\boxed{x = \frac{4}{9}}$$

Fun Fact:

$1/9 = 0.\overline{1}$ $5/9 = 0.\overline{5}$

$2/9 = 0.\overline{2}$ $6/9 = 0.\overline{6}$

$3/9 = 0.\overline{3}$ $7/9 = 0.\overline{7}$

$4/9 = 0.\overline{4}$ $8/9 = 0.\overline{8}$

ex 2 Convert $0.\overline{17}$ to a rational number.

Let $x = 0.\overline{17}$ So, $100x = 17.1717\ldots$

$$
\begin{aligned}
100x &= 17.1717\ldots \\
-\quad 1x &= 0.1717\ldots \\
\hline
99x &= 17
\end{aligned}
$$

$$\boxed{x = 17/99}$$

ex 3 Convert $0.\overline{153}$ to a rational number.

Let $x = 0.153153\ldots$ So, $1000x = 153.153153\ldots$

$$
\begin{aligned}
1000x &= 153.153\ldots \\
-\quad 1x &= 0.153\ldots \\
\hline
999x &= 153
\end{aligned}
$$

$$\boxed{x = 153/999}$$

ex 4 Convert $0.3\overline{19}$ to a rational number.

Let $x = 0.31919\ldots$

$10x = 3.1919\ldots$

$1000x = 319.1919\ldots$

So,
$$\begin{aligned} 1000x &= 319.1919\ldots \\ -\quad 10x &= 3.1919\ldots \\ \hline 990x &= 316 \end{aligned}$$

$$\boxed{x = \frac{316}{990}} \quad \text{or} \quad \boxed{x = \frac{158}{495}}$$

can reduce by 2

* In general, let x = original #.

may not be necessary { Multiply by a power of 10 to move the decimal right before the repeating part.

Multiply by a power of 10 to move the decimal right after the repeating part.

Then subtract + solve for x!

ex 5 Convert $0.12\overline{57}$ to a rational #.

Let $x = 0.125757...$
$100x = 12.5757...$
$10{,}000x = 1257.5757...$

$$\begin{array}{rl} 10{,}000x = & 1257.5757... \\ - \quad 100x = & 12.5757... \\ \hline 9900x = & 1245 \end{array}$$

$$\boxed{x = \frac{1245}{9900} \text{ or } \frac{83}{660}}$$

* Note: If a/b is a repeating rational number in simplest form with $b \neq 0$ and $b > a$ (proper fraction) then the repetend has at most $b-1$ digits.

This is because there are at most $b-1$ possible remainders.

For ex: $\frac{1}{7}$ has possible remainders of 1, 2, 3, 4, 5, or 6.

end 7-3