# Section 3.3 Types of Statements



A statement that is always true is called a <u>tautology</u>.

ex: $6+3=9$

A statement that is always false is called a <u>self-contradiction</u>. (Sometimes just called a contradiction.)

ex: $6+3=4$

p: I will get a haircut.
~p: I will not get a haircut.

opposite of p column    or    and

| $p$ | $\sim p$ | $p \vee \sim p$ | $p \wedge \sim p$ |
|---|---|---|---|
| T | F | T | F |
| F | T | T | F |
| | | tautology | self-contradiction |

<u>If a truth table column is:</u>

- all Ts then it is a tautology
- all Fs then it is a self contradiction
- a mix of Ts and Fs then it's neither

<u>ex1</u> Determine if the following statement is a tautology (all T), self-contradiction (all F), or neither (mix of T and F).

a) $(p \to q) \vee \sim q$

| $p$ | $q$ | $p \to q$ | $\sim q$ | $(p \to q) \vee \sim q$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

copied from summary sheet

all T

$\boxed{\text{tautology}}$

b) $(p \vee q) \wedge (\sim p \to q)$

| $p$ | $q$ | $p \vee q$ | $\sim p$ | $\sim p \to q$ | $(p \vee q) \wedge (\sim p \to q)$ |
|---|---|---|---|---|---|
| T | T | T | F | T | T |
| T | F | T | F | T | T |
| F | T | T | T | T | T |
| F | F | F | T | F | F |

and

copied from summary sheet

$\boxed{\text{neither}}$

c) $(p \leftrightarrow q) \wedge (\sim p \leftrightarrow \sim q)$

| $p$ | $q$ | $p \leftrightarrow q$ | $\sim p$ | $\sim q$ | $\sim p \leftrightarrow \sim q$ | $(p \leftrightarrow q) \wedge (\sim p \leftrightarrow \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T |
| T | F | F | F | T | F | F |
| F | T | F | T | F | F | F |
| F | F | T | T | T | T | T |

and

copied from summary sheet

mix of T and F

$\boxed{\text{neither}}$

Two compound statements that have identical truth values are said to be logically equivalent.

Two compound statements that have truth values that are exactly opposite are said to be negations.

ex 2 Determine if the 2 statements are logically equivalent, negations, or neither.

$$\sim(p \vee q)\ ;$$
$$p \rightarrow \sim q$$

| $p$ | $q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim q$ | $p \rightarrow \sim q$ |
|---|---|---|---|---|---|
| T | T | T | F | F | F |
| T | F | T | F | T | T |
| F | T | T | F | F | T |
| F | F | F | T | T | T |

copied from summary sheet

neither

De Morgan's Laws for sets:

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

We have a similar version for logic.

$$\sim(p \wedge q) = \sim p \vee \sim q$$
$$\sim(p \vee q) = \sim p \wedge \sim q$$

ex 3 Use De Morgan's Laws to write the negation of the statement.

a) I will study or I will nap.

p: I will study.
q: I will nap.

$p \vee q$

Negation is: $\sim(p \vee q) \underset{\uparrow}{=} \sim p \wedge \sim q$

De Morgan's Laws

I will not study and I will not nap.

b) It is raining and I do not have an umbrella.

p: It is raining
q: I do not have an umbrella.

$p \wedge q$

Negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$

De Morgan's Laws

$\boxed{\text{It is not raining or I do have an umbrella.}}$

There are 3 logical variants of $p \to q$

1. converse $q \to p$ flips order
2. inverse $\sim p \to \sim q$ negates both
3. contrapositive $\sim q \to \sim p$ flips orders and negates

* The converse and inverse are logically equivalent.

* The original and contrapositive are logically equivalent.

ex 5 Write the converse, inverse, and contrapositive for:

$$\sim p \to (q \wedge p)$$

Converse: flips order $\boxed{(q \wedge p) \to \sim p}$

simplify

Inverse: negates both $\boxed{\sim(\sim p)} \to \sim(q \wedge p)$

$\boxed{p \to \sim(q \wedge p)}$

Contrapositive: flips order and negates both

$\sim(q \wedge p) \to \boxed{\sim(\sim p)}$ $\boxed{\sim(q \wedge p) \to p}$

simplify